Generalized beams in $ABCDGH$ systems

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Abstract

For a generalized beam at the source plane passing through co-located aperture and a propagation path consisting of an off-axis $x$–$y$ asymmetric $ABCDGH$ system, the receiver plane irradiance expression is derived using the Collins integral. A collection of source beam profiles that are obtainable from the generalized beam formulation are illustrated. Plots are given for viewing the progress of selected generalized beam types along the propagation axis, containing a single thin lens, co-centric and misaligned in the $x$-direction. The received power falling onto a finite aperture surface is calculated for various misalignment situations.

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1. Introduction

The expression of Huygens Fresnel diffraction integral in terms of $ABCDGH$ matrix elements was initiated by Collins [1], hence bearing his name. Since then, $ABCDGH$ matrix formulation has become a versatile tool for studying the propagation characteristics of beams in free space. More popularly used version is the reduced $ABCD$ system, where $G$ and $H$ elements of the matrix are eliminated, if there are no misalignments of optical items encountered on the propagation paths, or no axes transformations are envisaged.

Cai and Lin have made several contributions in this area by investigating the propagation properties of beams with elliptical flat-topped profiles [2], hollow elliptical Gaussian beams [3] and partially coherent elliptical flattened Gaussian beams [4] for both aligned and misaligned optical systems. A series of papers were published [5–8] in which $ABCDGH$ matrices for a wide variety of optical components and propagation environments were derived and listed along with the application of the diffraction integral for $ABCDGH$ matrix arrangements. Further studies were conducted for the cases of specialized beams such as Bessel–Gaussian [9], Hermite–Laguerre–Gaussian [10] and Gaussian-Schell model [11] beams passing through $ABCD$ matrices. The effects of tilt and jitter of on-path optical elements within the context of $ABCD$ ray matrices were investigated in [12,13]. $ABCD$ transfer matrix was applied in [14] to study the propagation of elegant Hermite-cosine beams for unapertured and apertured cases. $ABCD$ systems were also used for the treatment of off-axial beams [15]. It was shown in [16] that $ABCDGH$ diffraction integral is also valid for a curved optical axis.

In the more general sense, there are two methods to treat the propagation of a light beam in a misaligned optical system, one is $ABCDGH$, another is $ABCDGFH$ [1,3]. Here it is the former that we study. Recently, we presented [17] results on the investigation of Hermite hyperbolic/sinusoidal beams propagating in $ABCD$ systems containing $x$–$y$ asymmetry. In this paper, we make the following extensions to our earlier work [17]:

(a) Via the concept of generalized beam, ability to care for a wide range properties and selection of beams under one single formulation,
(b) the use of ABCDGH matrix instead of the conventional ABCD, thus the capability to account for optical component and axes misalignments,

d) calculation of power falling onto an aperture placed on the receiver plane and its variation against optical component misalignments.

2. Formulation

The source field expression of the generalized beam is [18]:

\[ u_s(s) = u_s(s_x, s_y) = \sum_{i=1}^{N} \sum_{a,m} A_{i\alpha m} \exp(-i\theta_{i\alpha m}) \mathcal{H}_x(a_{i\alpha m}s_x + b_{i\alpha m}) \times \exp\left[-\left(0.5k_s x_{i\alpha m}^2 + i V_{i\alpha m}s_y\right)\right] \times \exp\left[-\left(0.5k_s y_{i\alpha m}^2 + i V_{i\alpha m}s_y\right)\right], \]  

(1)

where \((s_x, s_y)\) designates the decomposition of the vector \(s\) into \(x\) and \(y\) components. \(A_{i\alpha m}\) and \(\theta_{i\alpha m}\) are, respectively the amplitude and the phase of the \((i\alpha m)\) component of the source field, \(H_x(a_{i\alpha m}s_x + b_{i\alpha m})\) and \(H_m(a_{i\alpha m}s_y + b_{i\alpha m})\) are Hermite polynomials defining the beam distribution for \(s_x\) and \(s_y\) directions, where \(n\) and \(m\) are the order, \(a_{i\alpha m}\) and \(a_{i\alpha m}\) stand for the width, \(b_{i\alpha m}\) and \(b_{i\alpha m}\) are the complex displacement parameters,

\[ x_{i\alpha m} = 1/(k x_{i\alpha m}^2) + i/F_{i\alpha m}, \quad y_{i\alpha m} = 1/(k y_{i\alpha m}^2) + i/F_{i\alpha m}. \]  

(2)

Here \(x_{i\alpha m}\) and \(y_{i\alpha m}\) are Gaussian source sizes, \(F_{i\alpha m}\) and \(F_{i\alpha m}\) are the source focusing parameters along \(s_x\) and \(s_y\) directions, \(k = 2\pi/\lambda\) is the wave number with \(\lambda\) being the wavelength and \(i = (-1)^{0.5}\). \(V_{i\alpha m}\) and \(V_{i\alpha m}\) are the complex parameters used to create physical location displacement and phase rotation for the source field or a combination of both [19]. Furthermore by appropriately setting them as purely real or imaginary quantities, and implementing a summation over two terms, i.e. \(N = 2\), we are successively able to attain sinusoidal and hyperbolic Gaussian beams discussed earlier [17].

The source field described by Eq. (1), after traveling in the free space environment containing optical items, whose collection is represented by the optical box, will reach the receiver plane located at an axial distance of \(L\). Fig. 1 illustrates the associated geometry. With the help of Collins integral, the field at the receiver plane, \(u_r(p) = u_r(p_x, p_y)\), is attained [1,7] in the way shown below

\[ u_r(p) = u_r(p_x, p_y) = \frac{-i}{\pi} \exp\left\{-0.5i[H_x(B_s G_s - A_s H_s)/B_s + H_y(B_s G_y - A_s H_y)/B_y]/k\right\} \times \exp\left\{i(0.5k D p_x^2 + H p_x)/B_s\right\} \times \exp\left\{i(0.5k D p_y^2 + H p_y)/B_y\right\}/(2\pi B_y^{0.5} B_y^{0.5}) \times \int_{t_1}^{t_2} \int_{t_1}^{t_2} d^2 s_u(s) \exp\left[0.5ik(A_s^2 x_s^2/B_s + A_s^2 y_s^2/B_y)/B_s\right] \times \exp\left\{i[B_s G_s - A_s H_s - kp_x] s_x/B_s + (B_s G_y - A_s H_y - kp_y) s_y/B_y]\right\}. \]  

As understood from Fig. 1, the positional vector on the receiver plane, also decomposed into \(p = (p_x, p_y)\). The terms, \(t_1, t_2, t_1, t_2\) appearing in the integral limits of Eq. (3) for \(s_x\) and \(s_y\) directions, refer to the dimensions of a rectangular aperture that may be placed on an arbitrary position of the source plane. The parameters \(A_s, A_y, B_s, B_y, D_s, D_y, G_s, G_y, H_s\) and \(H_y\) are the respective \(x\) and \(y\) elements of the ABCDGH matrix defining the optical content of the involved propagation geometry, such that if for example the box illustrated in Fig. 1 were to be a single thin lens, misaligned in the \(x\)-direction by an amount, \(x_0\), having a tilt of \(\theta_x\) with respect to \(x\)-axis, and this lens was characterized by the parameters \(x_{ex}\) and \(x_{ey}\), then the following matrices would hold for the entire link from source to receiver

\[ \begin{bmatrix} A_s & B_s & 0 & 0 \\ C_s & D_s & 0 & 0 \\ G_s & H_s & 1 & 0 \\ \end{bmatrix} = \begin{bmatrix} 1 + i x_{ex} \cos \theta_x & L_1 + L_2(1 + i x_{ex} \cos \theta_x L_1) & 0 \\ i x_{ex} \cos \theta_x & 1 + i x_{ex} \cos \theta_x L_1 & 0 \\ 0 & 0 & 1 \\ \end{bmatrix} \]  

(4)

\[ \begin{bmatrix} A_y & B_y & 0 & 0 \\ C_y & D_y & 0 & 0 \\ G_y & H_y & 1 & 0 \\ \end{bmatrix} = \begin{bmatrix} 1 + i x_{ey} L_2 \cos \theta_y & L_1 + L_2(1 + i x_{ey} L_1 / \cos \theta_y) & 0 \\ i x_{ey} \cos \theta_y & 1 + i x_{ey} L_1 \cos \theta_y & 0 \\ 0 & 0 & 1 \\ \end{bmatrix} \]  

(5)

The ABCDGH approach is able to account for any number of cascaded sections with different compositions. Thus, Eqs. (4) and (5) are obtained by multiplying the individual ABCDGH matrix representation of each section, namely \(L_1\), the lens and \(L_2\), in the reverse order [20]. The parameters \(x_{ex}\) and correspondingly \(x_{ey}\) are related to effective transmission apertures, \(x_{ex}\) and \(x_{ey}\) and focal lengths, \(F_{ex}\) and \(F_{ey}\), of the thin lens via

\[ x_{ex} = 1/(k x_{ex}^2) + i/F_{ex}, \quad x_{ey} = 1/(k x_{ey}^2) + i/F_{ey}. \]  

(6)

To solve the Collins integral, first Eq. (1) is substituted for \(u_s(s)\) in Eq. (3), then all Hermite polynomials are expanded in series, subsequently a term by term integration based on the derivation steps outlined in [17] is performed, in the end, leading to the following result

\[ u_r(p) = -i \exp\left\{-0.5i[H_x(B_s G_s - A_s H_s)/B_s + H_y(B_s G_y - A_s H_y)/B_y]/k\right\} \times \exp\left\{i(0.5k D p_x^2 + H p_x)/B_s\right\} \times \exp\left\{i(0.5k D p_y^2 + H p_y)/B_y\right\}/(2\pi B_y^{0.5} B_y^{0.5}) \times \int_{t_1}^{t_2} \int_{t_1}^{t_2} d^2 s_u(s) \exp\left[0.5ik(A_s^2 x_s^2/B_s + A_s^2 y_s^2/B_y)/B_s\right] \times \exp\left\{i[B_s G_s - A_s H_s - kp_x] s_x/B_s + (B_s G_y - A_s H_y - kp_y) s_y/B_y]\right\}. \]  

(7)
where

\[
S_{jy} = \sum_{l_{y1}=0}^{\lfloor l_{y1} \rfloor} (-1)^{l_{y1}} 2^{-l_{y1}} T_{jy1} \left( \frac{n}{2l_{y1}} \right) \sum_{l_{x1}=0}^{\lfloor l_{x1} \rfloor} (b_{x1})^{l_{x1}} (iQ_{y1}/Q_{x1})^{l_{x1}} \left( \frac{1}{2} \right)^{l_{x1}!} x_{l_{x1}!} (2.5Q_{x1}/B_{x1})^{l_{y1}!} \\
\times \sum_{k_{x2}=0}^{\lfloor 0.5(k_{x2}-1) \rfloor} \binom{0.5(k_{x2}-1)}{k_{x2}} = 2^{k_{x2}}(k_{x2} - 1 - 2k_{x2})!! \\
\times \binom{g_{x1}^{-1}}{g_{x1}^{-1}} \text{exp} \left[ - \frac{0.5}{B_{x1}Q_{x1}} (t_{x1}Q_{x1} - iQ_{y1})^2 \right] \\
\times \frac{0.5}{B_{x1}Q_{x1}} (t_{x1}Q_{x1} - iQ_{y1})^2 \left( t_{x2}Q_{x2} - iQ_{y2} \right)^2 \\
- \frac{0.5}{B_{x1}Q_{x1}} (t_{x2}Q_{x2} - iQ_{y2})^2 \left( t_{x2}Q_{x2} - iQ_{y2} \right)^2 \\
- \frac{0.5}{B_{x1}Q_{x1}} (t_{x2}Q_{x2} - iQ_{y2})^2 \left( t_{x2}Q_{x2} - iQ_{y2} \right)^2 \\
\times \text{erf} \left[ \frac{0.5}{B_{x1}Q_{x1}} (t_{x2}Q_{x2} - iQ_{y2}) \right] \\
- \text{erf} \left[ \frac{0.5}{B_{x1}Q_{x1}} (t_{x2}Q_{x2} - iQ_{y2}) \right]
\]

From Eq. (8), \( S_{jy} \) is attained by changing all \( x \) subscripts to \( y \) and all \( n \) to \( m \), while the rest of the terms appearing in Eqs. (7) and (8) are defined as

\[
Q_{y1} = -B_{y1}V_{x1} + B_{y1}G_{y1} - A_{y1}H_{y1} - kp_{y1}, \\
Q_{y2} = -B_{y2}V_{x2} + B_{y2}G_{y2} - A_{y2}H_{y2} - kp_{y2}, \\
Q_{x1} = k(x_{x1}B_{x1} - iA_{x1}), \\
Q_{x2} = k(x_{x2}B_{x2} - iA_{x2}),
\]

\( g_{x1} \) and \( g_{x2} \) assume the values +1 or −1, subject to the following conditions,

\[
g_{x1} = +1, \quad g_{x2} = +1, \quad \text{when} \quad \text{real}(iQ_{y1}/Q_{x1}) \leq t_{x1} < t_{x2} \\
g_{x1} = -1, \quad g_{x2} = +1, \quad \text{when} \quad t_{x1} < \text{real}(iQ_{y1}/Q_{x1}) < t_{x2} \\
g_{x1} = -1, \quad g_{x2} = -1, \quad \text{when} \quad t_{x1} < t_{x2} \leq \text{real}(iQ_{y1}/Q_{x1})
\]

real(\( x \)) is the real part of \( x \), \( ! \) denotes the factorial, (\( k \)! \( = 1 \times 3 \times \cdots (k - 1) \) or \( k! = 2 \times 4 \times \cdots (k - 1) \) depending on whether \( k \) is odd or even. The square brackets appearing in the upper limit of some summations in Eq. (8) indicate that the integral part of the enclosing expression is to be taken. The operator, \( \text{rem}(k, 2) \) generates the remainder of \( k \) when divided by 2. The sign \( |x| \) implements the absolute value operation on \( x \). Binomial coefficients are shown in the forms of \( \binom{C_{1}}{C_{2}} \) such that, \( \binom{C_{1}}{C_{2}} = C_{1}/[(C_{1} - C_{2})!C_{2}]! \), \( T_{x1} = 1 \times 3 \times \cdots \times (2l_{x1} - 1) \) for \( l_{x1} \neq 0 \), \( \text{erf}(.) \) is the error function.

Note that the terms contained inside the summations of Eq. (7) are actually \( \ell \) dependent. But all throughout Eqs. (7)–(9), apart from those directly originating from the source field expression of Eq. (1), we have not included the letter \( \ell \) as a subscript for the other terms, in order to avoid the unnecessary complexity in the notation.

The \( G \) and \( H \) elements of the matrix merely stand for optical element displacements or misalignments of the axes [7], by setting them to zero, Eq. (7) will satisfactorily reduce to Eq. (6) of [17].

We find the irradiance at receiver plane by multiplying the receiver field with its conjugate, hence

\[
I_{r}(p) = u_{r}(p)u_{r}^{\ast}(p),
\]

where, \( \ast \) stands for the conjugate.

3. Results and discussions

Based on the numerical evaluation of Eqs. (1) and (11), this section provides the results in the form of irradiance
graphs against the variations of source, optical element and propagation parameters, where the two last items are largely embedded into the elements of the $ABCDGH$ matrix. For this purpose, the source field is converted into the irradiance expression via

$$I_s(s) = u_s(s)u_s^*(s).$$  \hspace{1cm} (12)

In order to smooth out the differences and obtain more intelligible illustrations, the following normalizations are established

$$I_{SN}(s) = I_s(s)/\max I_s(s),$$

$$I_{N}(p) = I_s(p)/\max I_s(s),$$  \hspace{1cm} (13)

where the operator max will furnish the peak value of the source irradiance.

In the accompanying graphs, the values of source and propagation parameters, the configuration settings of the aperture and the properties of the optical elements encountered in the propagation environment are written inside the box insets, usually placed above the related figures. Due to multitude of parameters, and space limitations however, only the essential ones are quoted. Additionally, in writing for the parameters $A_{mnm}$, $\theta_{mnm}$, $x_{sxnm}$, $x_{sxynm}$, $\nu_{sxnm}$ and $\nu_{sylnm}$, the subscripts $\ell, n$ and $m$ are omitted, and to establish $\ell$ indexing, including the mode indices $n$ and $m$, their values are written with spaces inserted in-between, such that these stated numerical values are assigned sequentially for $\ell = 1, 2, 3 \ldots$ so on. For the parameter values not stated, the governing rules will be that $x-y$ symmetry shall be assumed unless indicated otherwise, no aperture dimensions shall be given if it is not used, wavelength of operation, that is $\lambda$, shall always be 1.55 $\mu$m, source expressions are limited to maximum of three fields, with the focusing parameters, $F_{sxnm} = F_{sylnm} = 500$ m, being the same for all the three fields.

In the arguments of Hermite polynomials, the settings of $a_{sxnm} = 1/x_{sxnm}$, $a_{sylnm} = 1/x_{sylnm}$, $b_{sxnm} = b_{sylnm} = 0$ are used.

Eq. (1) is a general source field expression in the sense that enjoying the freedom of settings for many parameters, it is capable of generating all beam types previously investigated. In this manner, by arranging the source parameters according to the guidelines of Table 1 given in [21], it is possible to acquire from Eq. (1), pre-defined beams such as cos-Gaussian, cosh-Gaussian, sinh-Gaussian, annular Gaussian, pure Gaussian, flat-topped and their higher order counterparts. Additionally, Eq. (1) and subsequently Eq. (12) can be manipulated to create completely new and unusual beams that were not possible with earlier discrete formulations. Since the propagation characteristics of these pre-defined beams have already been investigated in our various publications (see for instance [17,18] and the references listed therein), in this study, it is the second type we shall concentrate on, namely those beams not obtainable with earlier discrete formulations. In this context, initially we demonstrate a range of source beams. Fig. 2 illustrates the 3D irradiance of a beam when the arrangements of source parameters are as shown in the inset of this picture. According to Fig. 2, this beam has an appearance akin to TEM$_{10}$. On the other hand, the examination against variations in $A_{nm}$, as done in Fig. 3, reveals that this particular beam is able to rotate radially with changing ratios of $A_{nm}$. In Fig. 3, an actual TEM$_{10}$ with half the source size of the main beams is shown in the background for reference purposes. Upon adjusting the source parameters as given in Fig. 4, specifically by introducing a phase change in the second field, making its source size much smaller and adding a third field, another interesting beam is achieved, where there is a thin central part connecting the two side lobes of the beam. From the beam of Fig. 4, by raising the source size of the second field to those of the other fields, while lowering the phase factor of the second field to $\pi/2$, and keeping the rest of parameters the same, a
well-like beam is attained as shown in Fig. 5. This beam closely resembles the annular beams studied in [22]. The beam of Fig. 5 will start to partition towards individual lobes, when the source size of the second field along the $s_x$ direction is doubled and a phase shift of $\pi/2$ is assigned to the third field. The lower plot in Fig. 5 reflects this change. Finally within the class of source beams, we illustrate a selection of asymmetric types that are displayed successively in the four plots of Fig. 6 along with their associated parameter settings.

Owing to the nature of formulation offered by Eq. (1), numerous choices are possible. We will however limit the receiver intensities to those source beams, which have been displayed above. The propagation behavior of the upper source beam belonging to Fig. 5, is investigated in Fig. 7. Here the propagation environment contains no aperture, but has a misaligned lens with indicated properties. Firstly we observe that the complex displacements are purely real, that is $V_x$ and $V_y = 150 \text{ m}^{-1}$, hence an off-axis situation emerges. This way, equal phase quantities both in $x$ and
$y$-directions are added to the beam, making no contribution to the irradiance profile when on the source plane. But the second plot of Fig. 7 shows that, as the propagation takes place, the beam becomes forced to drift into the negative portion of the transverse plane along the slanted (diagonal) axis, because of the $x$-$y$ equivalence in the displacement parameters. The dependence of such drifts on $V_x$ and $V_y$ are discussed at length in [19]. The other events to be noted are that, due to the finite size of the effective transmission aperture, only part of the irradiance falling onto the lens is captured, as evident from the third plot of Fig. 7. From the fourth plot of Fig. 7, we notice that, since the receiver plane comes after the focal length of the lens, the image becomes rotated by $180^\circ$, while it is contracted and peak irradiance of the beam is greatly increased due to focusing action of the lens. Finally, the
fourth plot of Fig. 7 allows us to gain an insight on how the misalignment of the lens setup will vary the beam footprint. The next case examined is based on the use of the fourth plot of Fig. 6 as the source beam. By taking this beam, Fig. 8 provides the behavior in propagation, when a square aperture of the indicated dimensions is applied immediately at the source plane. From the second plot of Fig. 8, we observe that the aperture is narrow, therefore excessive spreading is

Fig. 7. Propagation view of generalized beam belonging to Fig. 5, unapertured case.

Fig. 8. Propagation view of generalized beam belonging to fourth plot of Fig. 6, apertured case.
caused as apparent from the third plot of the same figure. Another interesting observation concerning the third plot of Fig. 8 in connection to the second plot of the same figure is that, the third plot of Fig. 8 bears resemblance to the \( \sin^2 \) function. This is an expected phenomena, since after becoming subjected to the hard aperture cutting, the source beam is transformed into almost a square beam type, then the propagation channel implementing a Fourier transform action on this new shape, finally produces the \( \sin^2 \) function-like irradiance profile, as also evident from the tails present in the third plot of Fig. 8. Finally from the fourth plot of Fig. 8, we witness the role played by the negative values of the displacement parameter, in the dislocation of the image on the receiver plane.

Now we consider the effects of misalignments due to displacement parameter, \( x_0 \), and the rotation parameter \( \theta_x \). Initially, it should be pointed out that due to our conventions, the positive \( x_0 \) values will move the lens in the negative direction along \( p_x \)-axis, while the negative \( x_0 \) values will give rise to the opposite. Secondly, in order to visualize the consequences of rotation parameter \( \theta_x \), a lens with asymmetrical effective transmission aperture size should be opted for, because rotating a circular lens would not result in the irradiance pattern changes at the receiver plane. It should be obvious from Figs. 7,8 that since our receiver plane is always located after the focal length of the lens, the images impinging on the receiver plane will be inverted by 180°.

The propagation details of the specific beam to be analyzed are given in Fig. 9 for zero displacement and zero rotation. A sample view of this arrangement is offered in Fig. 10, where both a lens possessing symmetrical \( x \) and \( y \) transmission apertures and an ellipsoid lens are superimposed on the incident beam. This way, we are able to visualize the amount of irradiance that will be captured by the lens. Further away from the lens, on the receiver plane, it is possible to calculate the normalized power incident on an on-axis aperture of radius \( z_0 \) via the following formulation

\[
P_{rN} = 2\pi \int_{0}^{\infty} r I_r(p) dr \left[ 2\pi \int_{0}^{\infty} r I_s(s) ds \right].
\]  

(14)

The denominator in Eq. (14) ensures that the above mentioned normalization is carried out with respect to the source plane power. Of course this could be replaced by the receiver plane irradiance integral, since no power is lost throughout the free space propagation. We note from Eqs. (1) and (7) that \( I_s \) and \( I_r \) are in Cartesian coordinates, hence a conversion to polar coordinates is required. This may be done analytically or during numeric integration. In our calculation, the latter choice was resorted to.

The results of the evaluations based on integrations of Eq. (14) are displayed in Fig. 11 for \( z_0 = 2 \text{ cm} \). Firstly it is seen that given the range of parameters in Fig. 11 and the visualization of Fig. 10, it is possible to capture the source power up to 70% (solid line curve), when the circular lens is aligned at \( x_0 = 1.5 \text{ cm} \) away from the origin. At this point, as Fig. 10 demonstrates, the lens will be in a position to collect the more power of the incident beam than, for instance, being co-centric with the axis of propagation. Pushing the lens further, such that \( x_0 > 1.5 \text{ cm} \), will cover more of the incident beam, but in these cases, the image on the receiver plane begins to go out of the receiver.

![Fig. 9. Propagation view of generalized beam belonging to second plot of Fig. 6, aligned case.](image)
aperture area. Hence, all curves of Fig. 11 more or less face a rapid fall once $x_0$ becomes larger than 1.5 cm. A similar situation occurs, when $x_0$ assumes negative displacement values. But it should be realized that due to the geometric location of the beam incident on the lens, the peak of received power coincides on the side of $x_0$ being positive.

Fig. 11 also proves that, lenses with reduced transmission apertures, i.e. ellipsoid shape and asymmetric focusing parameter will cause less power to be incident on the receiving aperture.

It is clear that the results delivered by Eqs. (7) and (8) may also have been achieved by direct numerical integration of Eq. (3). Such a task was undertaken for constructing the irradiance plots found in Figs. 7–9. Here it was observed that both results were identical, but direct integration would take a much longer time than using the analytically derived
expressions of Eqs. (7) and (8). For instance regarding Fig. 7, the analytic formulation would generate the graph in four seconds, while direct numerical integration would execute the same job in one hour and twenty four minutes. The corresponding durations for Figs. 8 and 9 were, respectively three seconds and three and a half seconds for analytic formulation, twenty seven minutes and three hour and twenty six minutes for numerical integration. It should be mentioned that in the case of evaluating the error function erf(.) present in Eq. (8), great advantage of speed, accuracy and reliability is gained via the use of algorithm described in [23].

4. Conclusion

Features governing free space propagation of generalized beams are examined in the presence of ABCDGH optical system. The propagation environment involves an aperture at the source plane and misaligned lens arrangement on the path of propagation. Our investigations concentrate on the irradiance profiles and the optical power at the receiver plane. The proposed generalized beam formulation is able to account for quite a wide range of different beam types including the previously studied cases such as Hermite-sinusoidal-Gaussian, higher order annular, flat, off-axis and multimode. In the current study, our choice is limited to source types that have not already been covered, while making extensions towards the misalignment situations of on-path optical elements. Source type variations are achieved by manipulating amplitude, phase, mode index, source sizes and displacement parameters of individual fields comprising the summation in the general beam formulation. Using a collection of thus generated beams, irradiance profiles, at the exit of the source plane aperture, before and after the lens and on the image plane have been plotted. The dependence of these beam profiles on the source and propagation parameters including the on-axis optical element in terms of positioning, shaping and rotation is studied. In this context, several ABCDGH configurations are considered such as symmetrical source aperture, misaligned, ellipsoid and asymmetric lens system, and focusing parameters. Finally, the power captured by receiver aperture of a given size, has been calculated, and its variation against the misalignments parameters of the lens has been shown graphically. Through our numerical evaluations, we could correctly trace the well known behaviors such as the Fourier transform relation of the optical field in the far field, inversion of the field when the focusing parameter is kept smaller than the distance to the receiver plane, and the irradiance asymmetry obtained by adjusting the complex source displacement parameters.

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