Self-similarity in ultrafast nonlinear optics

Recent developments in nonlinear optics have led to the discovery of a new class of ultrashort pulse, the 'optical similariton'. Optical similaritons arise when the interaction of nonlinearity, dispersion and gain in a high-power fibre amplifier causes the shape of an arbitrary input pulse to converge asymptotically to a pulse whose shape is self-similar. In comparison with optical solitons, which rely on a delicate balance of nonlinearity and anomalous dispersion and which can become unstable with increasing intensity, similaritons are more robust at high pulse powers. The simplicity and widespread availability of the components needed to build a self-similar amplifier capable of producing optical similaritons provides a convenient experimental platform to explore the fundamental nature of dynamical self-similarity. Here, we provide an overview of self-similar pulse propagation and scaling in optical fibre amplifiers, and their use in the development of high-power ultrafast optical sources, pulse synthesis and all-optical pulse regeneration.

JOHN M. DUDLEY[1]*, CHRISTOPHE FINOT[2,3], DAVID J. RICHARDSON[2] AND GUY MILLOT[3]

[1]Département d'Optique P. M. Duffieux, Institut FEMTO-ST, UMR 6174 CNRS-Université de Franche-Comté, 25030 Besançon, France

[2]Optoelectronics Research Centre (ORC), University of Southampton, Southampton SO17 1BJ, UK

[3]Département Optique, Interaction Matière-Rayonnement, Institut CARNOT de Bourgogne (ICB), UMR 5209 CNRS-Université de Bourgogne, 21078 Dijon, France

*e-mail: john.dudley@univ-fcomte.fr

Many natural phenomena exhibit self-similarity, reproducing themselves on different temporal and/or spatial scales. Although similarity and scaling laws in physics have been studied since the time of Galileo[1], their application in the modern era dates to the early years of the twentieth century, with an influential correspondence in *Nature* initiated by Lord Rayleigh[2–5] and the development of formal dimensional analysis by Buckingham[6,7]. The fundamental premise of dimensional analysis is that physical laws should be independent of the particular choice of units (be they metres, miles, furlongs or light years), and that it must be possible to express them using dimensionless parameters. Dimensional analysis is particularly powerful in reducing the number of degrees of freedom needed to describe a particular physical system, and in providing a systematic procedure to derive scaling relations between the key parameters involved. It thus provides a general technique for analysing phenomena across very different fields of physics, and even Rayleigh's brief report[2] includes a remarkable variety of examples from the resolving power of an optical microscope to the acoustic properties of the aeolian harp.

The use of scaling and normalization are common in the mathematical analysis of physical problems, but the existence of universal laws governing self-similar scale invariance in a system has a more profound fundamental significance, as it reveals the presence of internal structure and symmetry[8]. The basic concept of similar triangles is of course very familiar, but more sophisticated examples of geometrical self-similarity are widespread and can be found in settings ranging from natural branching patterns and coastlines[9], to the nodal properties of complex networks such as the World Wide Web[10].

As well as these examples involving spatial geometry, self-similarity also occurs in many dynamical problems as a natural stage in the temporal evolution of a system from a particular initial state. One of the most famous illustrations of this type concerns the evolution of the radius of a blast wave of a nuclear explosion, first analysed by the British physicist G. I. Taylor in the 1940s[11]. Although a nuclear weapon is a very complex device, Taylor's insight was to realize that the huge energy release from the explosion would result in the formation of a spherical shock wave whose self-similar expansion could be described in terms of only four dimensional quantities: the elapsed time t, the time-dependent shock-wave radius $R(t)$, the ambient air density ρ and the energy released E.

The application of dimensional analysis to this problem seeks to combine these four quantities to form dimensionless 'similarity parameters', and it is easy to see here how they combine into one such parameter: $\theta = \rho R^5/Et^2$. It follows immediately that the blast-wave radius expands according to the scaling law $R(t) = \theta^{1/5}(Et^2/\rho)^{1/5}$, where the similarity variable θ plays the role of a proportionality constant. In fact, numerical computation yields a specific value for θ (approximately unity) and Taylor himself was able to use declassified images of the 1945 Trinity explosion to quantitatively confirm this scaling hypothesis[12].

SELF-SIMILAR DYNAMICS

The blast-wave example is one where simple dimensional analysis works particularly well, but more sophisticated methods also exist to determine self-similar solutions for more complex systems. Such formal similarity techniques extend the toolbox available to mathematical physicists, and are of particular importance in analysing nonlinear problems described by partial differential equations — well known to be notoriously difficult to solve exactly.

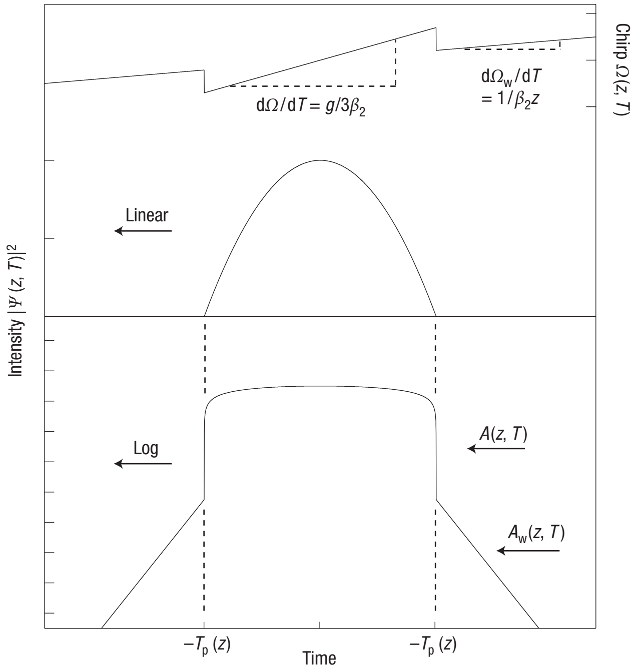

Figure 1 Graphical representation of generic similariton characteristics. Top: parabolic intensity profile (left axis) and linear chirp (right axis). Bottom: the intensity profile on a logarithmic scale.

For such cases, the approach is to reduce the degrees of freedom of the system by reformulating the problem in terms of similarity variables (where they exist) so that the original problem of solving partial differential equations can be recast into a simplified problem involving ordinary differential equations[13,14].

Although similarity techniques are common in fields such as hydrodynamics and plasma physics, their use in optics has not been widespread. Some significant studies have nonetheless been carried out, including research into the nonlinear dynamics of a range of lasers and optoelectronic devices[15–17], nonlinear self-action and collapse processes[18–22], stimulated Raman scattering[23], fractal structure excitation in soliton-supporting systems[24,25] and spatial fractal pattern formation[26–29]. Other studies with a materials emphasis have demonstrated self-similarity in the growth of Hill gratings[30] and the evolution of self-written waveguides[31,32].

A particular area of optics research where self-similar dynamical effects have attracted much recent interest is the study of nonlinear pulse propagation in optical fibre amplifiers. Fibre amplifiers are key components in optical telecommunications systems and high-power ultrafast source development, but are generally configured to operate such that nonlinear effects are negligible. However, recent results have demonstrated a fundamentally new operating regime where nonlinear propagation is, in fact, exploited to generate a particular class of ultrashort parabolic pulse that evolves self-similarly as it is amplified[33–36]. As is often the case, optical systems provide convenient experimental testbeds with which to study physical processes of widespread interest, and the study of dynamical self-similarity in ultrafast optics is developing into an increasingly active field of research.

CHARACTERISTICS AND SCALING OF OPTICAL SIMILARITONS

High-power ultrashort-pulse propagation in optical fibres is well known to be associated with distortions and break-up effects due to the interaction of the fibre nonlinearity and dispersion[37]. Although in the anomalous-dispersion regime of a fibre these effects can balance and yield soliton propagation[38,39], fundamental soliton stability exists at only one particular power level, and propagation at higher power excites higher-order solitons, which are sensitive to perturbation and break-up through soliton fission[40,41]. High-power propagation in the normal-dispersion regime is also subject to instability through optical wave breaking that develops on the temporal pulse envelope after a characteristic distance that depends on the initial pulse shape[42,43]. These distortions limit the energy of ultrashort pulses that can propagate in an optical material, and are particularly detrimental for the development of high-gain fibre amplifiers.

In 1993, however, Anderson *et al.* showed that high-power pulse propagation in the normal-dispersion regime was not inevitably associated with optical wave breaking and, in fact, wave breaking could be completely avoided for a particular class of pulse possessing a parabolic intensity profile and a linear frequency chirp[44]. This key physical insight was followed up in 1996 by Tamura and Nakazawa[45], who showed numerically that ultrashort pulses injected into a normal-dispersion fibre amplifier seemed to naturally evolve towards the parabolic-pulse profile and, moreover, retained their parabolic shape even as they continued to be amplified to high power. Tamura and Nakazawa attempted to verify these results experimentally, but the available pulse diagnostic techniques did not allow the generation of parabolic pulses to be conclusively confirmed.

The unambiguous experimental observation of parabolic-pulse generation was first reported by Fermann *et al.*[33] in 2000 in an ytterbium-doped fibre amplifier with normal dispersion at 1.06 μm. These measurements were compelling because the use of the ultrashort-pulse measurement technique of frequency-resolved optical gating (FROG) invented during the 1990s provided experimental access to the complex field of the parabolic pulse[46]. As well as representing a major experimental advance, the results in Fermann *et al.* also applied theoretical analysis on the basis of symmetry reduction to the nonlinear Schrödinger equation (NLSE) with gain, formally demonstrating the self-similar nature of the generated parabolic pulses.

A major result of this analysis was to show that the self-similar parabolic pulse is a rigorously asymptotic solution to the NLSE with gain, representing a type of nonlinear 'attractor' towards which any arbitrarily shaped input pulse of given energy would converge with sufficient distance. By analogy with the well-known solitary-wave behaviour of solitons, these self-similar parabolic pulses have come to be known as similaritons[34]; in fact, solitons themselves can also be interpreted as an example of self-similarity[8]. Further application of similarity techniques also yielded a quantitative description of the important intermediate-asymptotic regime, where the fine structure due to the initial conditions has disappeared yet the system has not reached its asymptotic state. This was a particularly significant advance from a fundamental viewpoint as it demonstrated the presence of similarity characteristics on different scales in this system.

The analysis yields closed-form expressions of particular simplicity for the self-similar parabolic-pulse characteristics that we describe below making reference to Fig. 1 (ref. 35). The results are obtained assuming that a fibre amplifier can be described by the NLSE with gain expressed in the following (dimensional) form:

$$i\frac{\partial \Psi}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 \Psi}{\partial T^2} - \gamma|\Psi|^2\Psi + i\frac{g}{2}\Psi.$$

Here, $\Psi(z,T)$ ($W^{1/2}$) is the slowly varying pulse envelope in a frame comoving at the envelope group velocity, β_2 ($ps^2\,m^{-1}$)

and γ ($W^{-1}\,m^{-1}$) are the fibre dispersion and Kerr nonlinear coefficients respectively, and g (m^{-1}) is the amplifier-distributed gain coefficient. Figure 1 shows the temporal pulse characteristics on both linear and logarithmic scales, with the former illustrating the parabolic nature of the central core and the latter showing the presence of low-amplitude wings. Writing the field as $\Psi(z,T)=A(z,T)\exp(i\Phi(z,T))$ gives the corresponding closed-form analytic expressions in terms of the amplitude and phase A and Φ respectively. The analytic expression for the amplitude $A(z,T)$ and the phase $\Phi(z,T)$ of the asymptotic solution ($|T|\le T_p(z)$) is

$$\Psi(z,T)=A(z,T)\exp(i\Phi(z,T))$$

$$A(z,T)=A_0\exp\left(\frac{g}{3}z\right)\sqrt{1-\frac{T^2}{T_p^2(z)}}$$

$$\Phi(z,T)=\varphi_0+\frac{3\gamma A_0^2}{2g}\exp\left(\frac{2}{3}gz\right)-\frac{g}{6\beta_2}T^2$$

$$A_0=\frac{1}{2}\left(\frac{gU_{in}}{\sqrt{\gamma\beta_2/2}}\right)^{1/3}\quad T_p(z)=\frac{6\sqrt{\gamma\beta_2/2}}{g}A_0\exp\left(\frac{g}{3}z\right),$$

where U_{in} is the input pulse energy, and T_p and A_0 characterize the parabolic-pulse width and amplitude respectively. The analytic expression for the amplitude $A_w(z,T)$ and the phase $\Phi_w(z,T)$ of the intermediate-asymptotic solution ($|T|>T_p(z)$) is

$$\Psi(z,T)=A_w(z,T)\exp(i\Phi_w(z,T))$$

$$A_w(z,T)=\frac{A_{w0}}{\sqrt{z}}\exp\left(\frac{g}{2}z\right)\exp\left(-\Lambda\frac{|T|}{z}\right)$$

$$\Phi_w(z,T)=\varphi_{w0}+\frac{\beta_2\Lambda^2}{2z}-\frac{T^2}{2\beta_2 z},$$

φ_0 and φ_{w0} are arbitrary constants, and the intermediate-asymptotic scaling parameters Λ and A_{w0} are determined numerically for particular initial conditions. The instantaneous frequency chirp across the pulse envelope is $\Omega(z,T)=-\mathrm{d}\Phi(z,T)/\mathrm{d}T$. The corresponding spectrum in the asymptotic limit is also parabolic and possesses a quadratic spectral phase.

The asymptotic temporal characteristics correspond to a strictly parabolic-pulse core with compact support. Although presented here for the case of a constant gain, these characteristics are, in fact, also observed under more arbitrary conditions when the gain varies longitudinally along the amplifier length[34,47,48]. The asymptotic nature of the solution is reflected in the fact that the amplitude and width scaling (in both the time and frequency domains) depends only on the amplifier parameters and the input pulse energy, and is independent of the input pulse shape. We also note that the pulse chirp is independent of propagation distance, a property of particular significance for optical compressor design. In the intermediate-asymptotic regime, the reshaping of any arbitrary input pulse generates exponentially decaying low-amplitude wings, but the scaling of the intermediate characteristics with distance is more complex, and does depend on the exact input pulse shape used. The scaling constants in this case must be determined numerically. The intermediate-asymptotic spectral characteristics are also complex, exhibiting both deviations from the ideal parabolic form and complex oscillatory structure whose relative energy contribution decreases as the spectrum evolves towards the asymptotic limit[35].

Figure 2 illustrates explicitly the attractive nature of the asymptotic parabolic-pulse solution. Here, we show numerical

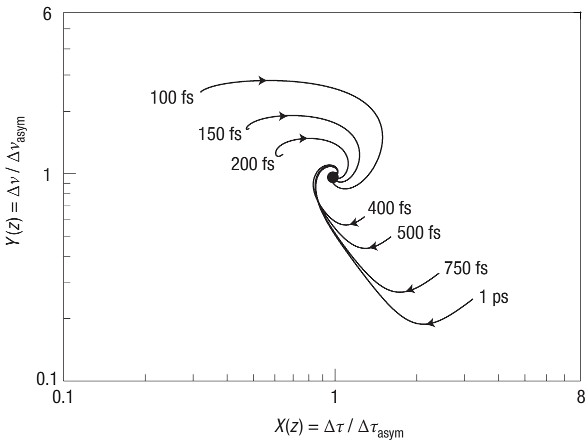

Figure 2 Phase-space portrait of different evolution trajectories of optical pulses in a self-similar amplifier. The results show the response of a 5-m normal-dispersion fibre amplifier to hyperbolic secant input pulses with durations (full-width at half-maximum) in the range 100 fs–1 ps. With fixed input pulse energy, all trajectories are attracted to the asymptotic sink. The phase-space variables X and Y are respectively calculated from the ratio of the evolving r.m.s. temporal and spectral widths relative to the corresponding r.m.s. widths of the expected asymptotic parabolic-pulse solution at the same distance. Specifically, for a propagation distance z, the r.m.s. temporal width is given by $\Delta\tau_{asym}(z)=(1/\sqrt{5})\sqrt{\gamma\beta_2/2}(6/g)A_0\exp(gz/3)$ and the r.m.s. spectral width is given by $2\pi\Delta\nu_{asym}(z)=(1/\sqrt{5})\sqrt{2\gamma/\beta_2}A_0\exp(gz/3)$.

simulation results calculating the evolution of 100-pJ input pulses with durations varying over the range 100 fs–1 ps in a 5 m amplifier with $\beta_2=+25\,ps^2\,km^{-1}$, $\gamma=5\,W^{-1}\,km^{-1}$ and $g=1.92\,m^{-1}$. The convergence to the asymptotic solution with propagation can be conveniently examined in a phase-space representation in terms of the ratios of the r.m.s. temporal and spectral widths relative to the widths expected for an asymptotic similariton pulse at the same distance (see the caption for definitions). Using this representation, we see that, although different input pulses do follow different evolution trajectories, they nonetheless are all attracted to the asymptotic parabolic-pulse solution with sufficient propagation.

Physically, the differences in the evolution trajectories arise because pulses of different duration experience different contributions from dispersive and nonlinear effects during the initial propagation phase. For practical purposes, it is important to optimally match the input pulse and amplifier parameters, and useful practical guidelines have been developed[35,49]. Studies have also examined the influence of the initial pulse shape on the evolution trajectory, and have shown that even quasi-rectangular supergaussian pulses evolve to the self-similar parabolic regime with sufficient propagation[35]. Significantly, all of the energy of the input pulse is retained in the asymptotic solution, in contrast to soliton propagation where dispersive wave radiation is shed from non-ideal input pulses[37].

The universality of the attractive evolution towards the self-similar profile has also recently been confirmed through theoretical analysis[50]. Related numerical studies have also clarified that although the asymptotic solution always corresponds to a linearly chirped parabolic similariton, parabolic pulses and similaritons are not always synonyms, because an injected parabolic pulse without the appropriate initial chirp will undergo non-self-similar evolution and follow its own particular trajectory into the asymptotic regime[49].

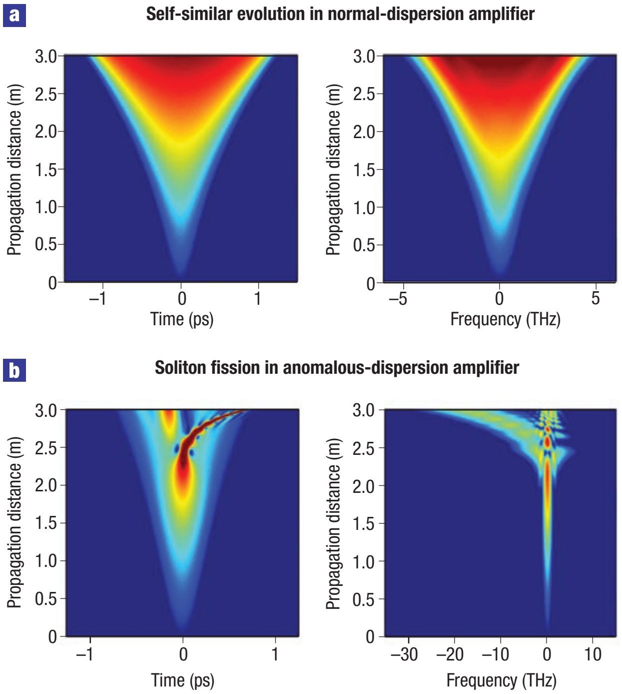

Figure 3 Simulated pulse evolution of propagation in fibre amplifiers comparing self-similar evolution with soliton fission. **a**,**b**, Normal (**a**) and anomalous (**b**) dispersion using parameters typical of realistic Yb- and Er-doped gain media respectively. The figures are false-colour representations (on a logarithmic scale) of the pulse temporal intensity and power spectrum. The dispersion, nonlinearity and gain are **a**, $\beta_2 = +25\ \mathrm{ps^2\ km^{-1}}$, $\gamma = 5\ \mathrm{W^{-1}\ km^{-1}}$, $g = 1.92\ \mathrm{m^{-1}}$ (integrated gain 25 dB) and **b**, $\beta_2 = -25\ \mathrm{ps^2\ km^{-1}}$, $\gamma = 5\ \mathrm{W^{-1}\ km^{-1}}$, $g = 1.54\ \mathrm{m^{-1}}$ (integrated gain 20 dB). The input pulse had a hyperbolic secant profile and 12-pJ energy. These results illustrate both the fundamental properties and practical advantages of self-similar amplification.

In contrast to other forms of nonlinear fibre propagation, a feature of the self-similar regime is that both the temporal and spectral width increase exponentially and monotonically with propagation. Figure 3 shows this explicitly, using numerical simulations to compare the evolution of a 300-fs pulse in fibre amplifiers with normal (Fig. 3a) and anomalous (Fig. 3b) dispersion. The simulations are based on the NLSE with gain and the addition of stimulated Raman scattering in the fused-silica host material to model realistic perturbative effects[37]. The parameters are given in the caption and correspond to realistic current technology.

The figure clearly shows the difference between amplification with normal and anomalous dispersion. With normal dispersion and self-similar dynamics, amplification is associated with the simultaneous increase in both temporal and spectral widths and the absence of any wave breaking or pulse distortion. In contrast, for an amplifier with the same gain yet with anomalous dispersion, as the pulse energy increases with amplification, instabilities become apparent and the pulse breaks up owing to the effect of soliton fission. In this case, both the pulse temporal and spectral structure is complex and the output characteristics would be undesirable for many applications.

Note that the soliton fission observed in the anomalous-dispersion regime is induced by stimulated Raman scattering, but this perturbation has negligible effect on the evolution in the normal-dispersion regime where soliton fission cannot occur. Indeed, it is even possible to exploit Raman scattering in fibres as a gain mechanism to generate parabolic pulses[51,52]. Nonetheless, under certain conditions, Raman and other effects can perturb the self-similar propagation, and studies have been carried out to determine the practical limitations of self-similar amplification. The results indicate that, although pulse shape fluctuations are generally negligible[53], Raman gain and the effect of the finite bandwidth of the amplifying transition can impair the amplified pulse quality and limit achievable pulse energies[54–57]. In addition, whereas third-order dispersion effects are also often negligible[58], they can be significant for the broadest spectra, although efficient compensation techniques have recently been developed[59].

EXPERIMENTAL ASYMPTOTICS

We consider the practical application of self-similar amplifiers more in the next section, but at this stage we describe experiments where we have examined their fundamental scaling properties using amplifiers operating around the telecommunications wavelength of 1,550 nm. Particular aims of these experiments were to develop convenient self-similar amplifier configurations using only readily available and relatively inexpensive fibre components, and to use a high-dynamic-range FROG set-up for detailed characterization.

In one experiment using a self-similar amplifier based on Raman gain in fused silica, the asymptotic nature of the generated parabolic pulses was explicitly verified by injecting a range of different input pulses, and examining the output pulses in each case[60]. The measured output pulse characteristics were invariant with input pulse duration and profile, being determined only by the amplifier parameters and input pulse energy. In another experiment, the evolution towards the asymptotic regime was explicitly examined in an erbium fibre amplifier[61]. Figure 4a shows simulations illustrating the expected amplification and reshaping of a 1.4-ps hyperbolic secant input pulse to a parabolic similariton for this system when operated at a gain of 13.6 dB. Corresponding experiments were then carried out by constructing such an amplifier and cutting it back in 50-cm segments to directly measure the pulse evolution. A feature of these results of particular experimental interest is the characterization of the pulse electric field over an intensity dynamic range of 40 dB, and with the possibility to resolve sub-100-fs structure over a 20-ps timebase. The measurements are amongst some of the most demanding ever made in ultrafast optics, and the results shown in Fig. 4b are in remarkable agreement with simulations. At the intermediate distance of 7 m, Fig. 4c compares experiment and simulation on a logarithmic scale to explicitly show the presence of intermediate-asymptotic wings about the central parabolic-pulse core. Further experiments have shown how the relative energy in the wings decreases as the pulse enters the asymptotic regime, and represent what is, to our knowledge, the first experimental observation of intermediate-asymptotic self-similar dynamics in optics[61].

PRACTICAL SELF-SIMILAR AMPLIFIERS

In parallel with the fundamentally oriented studies described above, the scaling properties of parabolic pulses have been applied to the development of a new generation of optical fibre amplifier. From a technological viewpoint, self-similar amplifiers possess a number of very attractive features. In common with the well-known technique of chirped-pulse amplification (CPA), catastrophic pulse break-up due to excessive nonlinear phase shifts is avoided[62]. However, in contrast to CPA where the aim is to avoid nonlinearity by dispersive pre-stretching before amplification, a self-similar amplifier actively exploits nonlinearity, allowing for the very attractive possibility of obtaining output pulses after recompression that are actually shorter than the initial input pulse.

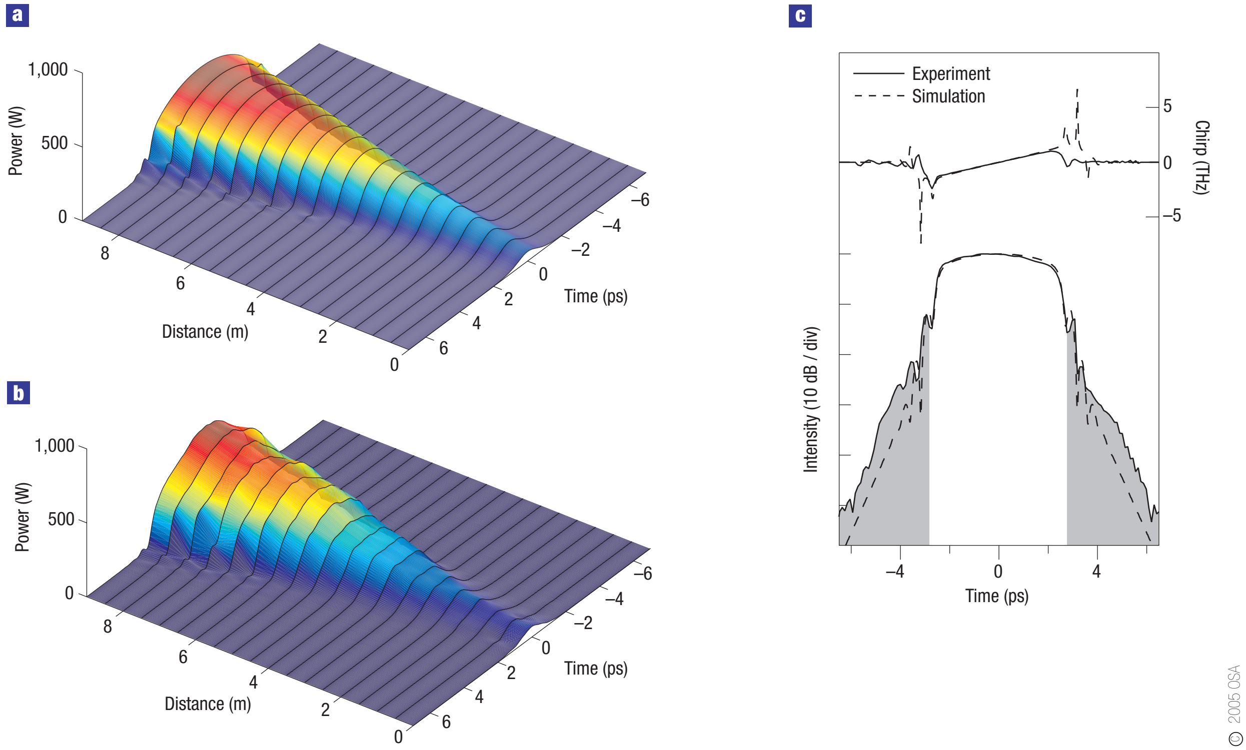

Figure 4 Similariton evolution in a 9-m-long erbium fibre amplifier. **a**,**b**, Simulation (**a**) and experimental (**b**) results. **c**, Detail of the similariton intensity and chirp after 7-m propagation. The shading distinguishes the intermediate-asymptotic wings from the parabolic similariton core. Adapted from ref. 61.

Although gain-bandwidth limitations typically perturb self-similar spectral broadening at pulse energies exceeding the microjoule level, similariton amplifiers present a convenient alternative to more complex CPA systems below this limit[55]. Moreover, the existence of analytic design criteria for self-similar amplifiers makes it straightforward to tailor system design to a wide range of input pulses and amplifier types. Specifically, the fact that the asymptotic pulse duration and chirp depend only on the input pulse energy makes the amplification process insensitive to a wide class of seed-pulse instabilities. As a result, practical self-similar amplifiers have been demonstrated using ytterbium, erbium and Raman gain media, seed pulses in the range 180 fs–10 ps, fibre lengths in the range 1.2 m–5.3 km and gains varying from 14–32 dB (refs 33,51,53,63–66). The possibility to obtain amplified pulse energies exceeding 1 μJ in an environmentally stable and polarization-maintaining configuration has been a significant recent demonstration[67]. Another important result has been the use of passively mode-locked vertical-external-cavity surface-emitting semiconductor lasers as the primary master-oscillator seed-pulse source[68].

The fact that the output pulse chirp depends only on the amplifier gain and dispersion considerably simplifies the post-compressor design, and high-quality compressed pulses in the 100-fs regime with megawatt peak powers have been obtained[65,67]. An exciting development has been the application of the novel dispersion properties of photonic bandgap optical fibre[69] to replace the use of bulk gratings in the compression stage, allowing the realization of an all-fibre format source of ~200-fs pulses around 1,550 nm (ref. 61). Figure 5 shows results obtained with such a system, showing an electron micrograph of a typical bandgap-fibre used (Fig. 5a) and the high-quality compressed pulses obtained (Fig. 5b). A more recent experiment has developed this system further, combining self-similar dynamics with higher-order soliton propagation to develop a hybrid similariton–soliton system yielding pulses as short as 20 fs (ref. 70). At the operating wavelength of 1,550 nm, this pulse duration represents only four optical cycles, and FROG characterization enables the reconstruction of the electric field to explicitly illustrate their few-cycle nature as shown in Fig. 5c.

THE 'SIMILARITON LASER'

The development of any new optical amplifier technology invariably suggests application in a laser, and self-similar dynamics are no exception. The combination of self-similar propagation with optical feedback has been studied both theoretically and experimentally, and promises a new generation of fibre lasers that overcome existing power limitations of soliton mode-locking[71–76]. Although self-similar evolution naturally presents many advantages in the design of fibre lasers, the possibility of observing self-similar dynamics in solid-state mode-locked lasers such as Ti:sapphire has also been considered[77]. Significantly, this has motivated work on exploiting normal-group-velocity-dispersion propagation dynamics to extend solid-state oscillator systems to the microjoule regime, and on more general considerations of pulse-shaping mechanisms in this regime[78–80].

GHz SIMILARITONS

In parallel with research into the generation of high-power ultrashort pulses, self-similar propagation effects have also been studied at the much lower powers associated with telecommunications systems. The interest here is not so much in the power scaling, but in the use of the unique

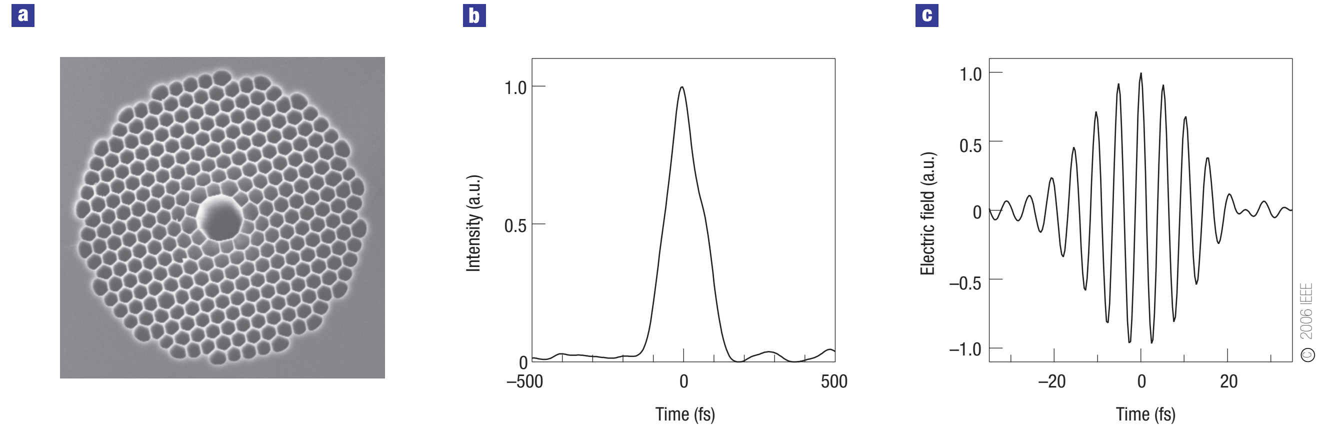

Figure 5 Few-cycle pulse generation around 1,550 nm through the combination of nonlinear similariton and soliton dynamics. **a**, Electron micrograph of a typical hollow-core photonic bandgap fibre suitable for linear chirp compensation around 1,550 nm. **b**, High-quality pulses of around 200-fs duration after using this fibre to compress the linearly chirped parabolic similaritons generated in an erbium-doped fibre amplifier. **c**, Further nonlinear compression of the pulses to the few-cycle regime by exploiting nonlinear soliton compression to obtain 20-fs pulses, representing around four optical cycles at this wavelength. Part **c** adapted from ref. 70.

asymptotic properties of high-repetition-rate parabolic pulses for manipulation and shaping applications. The crucial first step here was the generation of GHz-repetition-rate parabolic-pulse trains around 1,550 nm, and the demonstration of their potential for low-noise multiwavelength source development[53]. This research has since opened the door to applying parabolic-pulse technology for the ultrafast signal processing applications that are essential for an all-optical network.

For example, the invariance of the output profile to input pulse fluctuations is of great interest for pulse-shaping and pulse-synthesis applications, enabling highly stable output that can be applied to spectral slicing[81,82], for example. Applications in the domain of regeneration and retiming are also promising and can benefit from the perfectly linear chirp induced through self-phase or cross-phase modulation[83–85]. A further important development has been the demonstration of passive parabolic-pulse generation in undoped dispersion-decreasing fibre[86,87], although the effect of third-order dispersion in this case has been shown to have a more limiting effect than in an amplifying fibre[88]. In addition, the wider study of self-similar dynamics in fibres with longitudinally varying parameters is becoming an active field of research relevant to pulse shaping and pulse compression[89–93], linked to previous studies of certain classes of soliton propagation[94,95].

OUTLOOK

It is clear that nonlinear optical fibre amplifiers that exploit self-similar evolution dynamics have developed into a mature alternative to other ultrafast pulse generation and shaping techniques. In parallel, the success of self-similarity analysis in nonlinear fibre optics is motivating more general studies into the dynamics of guided wave propagation[96–98] as well as related self-similar evolution in physical systems such as Bose–Einstein condensates[99,100].

From a more general perspective, it is also interesting to note that current research in many areas of nonlinear photonics increasingly relies on sophisticated simulation and modelling and, although such numerical treatments are indispensable, the underlying physics is sometimes difficult to readily interpret. In our opinion, one of the most attractive features of the extensive recent interest in self-similarity is that it has reminded the nonlinear-optics community of an extensive array of mathematical tools that can be used to find analytic solutions to complex dynamical problems. As mode-locked lasers generate ever-shorter and more intense pulses, such nonlinear propagation problems will become of increasing importance, and a complete understanding of the propagation dynamics will be required both for source optimization and the many potential multidisciplinary applications. To this end, we believe that the search for universal patterns in these phenomena, as pioneered centuries ago by Galileo, will remain a very profitable direction of research.

doi:10.1038/nphys705

References

1. Galileo, G. *Dialogues Concerning Two New Sciences* (Crew, H. & de Salvio, A. translators) 1–5 (MacMillan, New York, 1914).
2. Lord Rayleigh, The principle of similitude. *Nature* **95**, 66–68 (1915).
3. Riabouchinsky, D. *Nature* **95**, 591 (1915).
4. Lord Rayleigh, *Nature* **95**, 644 (1915).
5. Buckingham, E. The principle of similitude. *Nature* **96**, 396–397 (1915).
6. Buckingham, E. On physically similar systems; illustrations of the use of dimensional equations. *Phys. Rev.* **4**, 345–376 (1914).
7. Macagno, E. O. Historico-critical review of dimensional analysis. *J. Franklin Inst.* **292**, 391–402 (1971).
8. Barenblatt, G. I. *Scaling, Self-Similarity, and Intermediate Asymptotics* (Cambridge Univ. Press, Cambridge, 1996).
9. Mandelbrot, B. *The Fractal Geometry of Nature* (Freeman, San Francisco, 1982).
10. Song, C., Havlin, S. & Makse, H. A. Self-similarity of complex networks. *Nature* **433**, 392–394 (2005).
11. Taylor, G. I. The formation of a blast wave by a very intense explosion. I: Theoretical discussion. *Proc. R. Soc. Lond. A* **201**, 159–174 (1950).
12. Taylor, G. I. The formation of a blast wave by a very intense explosion. II: The atomic explosion of 1945. *Proc. R. Soc. Lond. A* **201**, 175–186 (1950).
13. Olver, P. J. *Applications of Lie Groups to Differential Equations* 2nd edn (Graduate Texts in Mathematics, Vol. 107, Springer, New York, 1993).
14. Sachdev, P. L. *Self-Similarity and Beyond: Exact Solutions of Nonlinear Problems* (CRC Monographs and Surveys in Pure and Applied Mathematics, Chapman and Hall, London, 2000).
15. Ikeda, K. Multiple-valued stationary state and its instability of the transmitted light by a ring cavity system. *Opt. Commun.* **30**, 257–261 (1979).
16. Uchida, A., Rogister, F., Garcia-Ojalvo, J. & Roy, R. Synchronization and communication with chaotic laser systems. *Prog. Opt.* **48**, 203–341 (2005).
17. Argyris, A. *et al.* Chaos-based communications at high bit rates using commercial fibre-optic links. *Nature* **437**, 343–346 (2005).
18. Afanas'ev, A. A., Kruglov, V. I., Samson, B. A., Jakyte, R. & Volkov, V. M. Self-action of counterpropagating axially symmetrical light beams in a transparent cubic-nonlinearity medium. *J. Mod. Opt.* **38**, 1189–1202 (1991).
19. Kruglov, V. I., Logvin, Y. A. & Volkov, V. M. The theory of spiral laser-beams in nonlinear media. *J. Mod. Opt.* **39**, 2277–2291 (1992).
20. Dyachenko, S., Newell, A. C., Pushkarev, A. & Zakharov, V. E. Optical turbulence: Weak turbulence, condensates and collapsing filaments in the nonlinear Schrödinger equation. *Physica D* **57**, 96–160 (1992).
21. Moll, K. D., Gaeta, A. L. & Fibich, G. Self-similar optical wave collapse: Observation of the Townes profile. *Phys. Rev. Lett.* **90**, 203902 (2003).

22. Gaeta, A. L. Collapsing light really shines. *Science* **301**, 54–55 (2003).
23. Menyuk, C. R., Levi, D. & Winternitz, P. Self similarity in transient stimulated Raman scattering. *Phys. Rev. Lett.* **69**, 3048–3051 (1992).
24. Soljacic, M., Segev, M. & Menyuk, C. R. Self-similarity and fractals in soliton-supporting systems. *Phys. Rev. E* **61**, R1048–R1051 (2000).
25. Sears, S., Soljacic, M., Segev, M., Krylov, D. & Bergman, K. Cantor set fractals from solitons. *Phys. Rev. Lett.* **84**, 1902–1905 (2000).
26. Karman, G. P., McDonald, G. S., New, G. H. C. & Woerdman, J. P. Laser optics: Fractal modes in unstable resonators. *Nature* **402**, 138 (1999).
27. Berry, M. V., Marzoli, I. & Schleich, W. P. Quantum carpets, carpets of light. *Phys. World* **14**, 39–44 (2001).
28. Courtial, J., Leach, J. & Padgett, M. J. Fractals in pixellated video feedback. *Nature* **414**, 864 (2001).
29. Huang, J. G. & McDonald, G. S. Spontaneous optical fractal pattern formation. *Phys. Rev. Lett.* **94**, 174101 (2005).
30. An, S. & Sipe, J. E. Universality in the dynamics of phase grating formation in optical fibers. *Opt. Lett.* **16**, 1478–1480 (1991).
31. Kewitsch, A. S. & Yariv, A. Self-focusing and self-trapping of optical beams upon photopolymerization. *Opt. Lett.* **21**, 24–26 (1996).
32. Monro, T. M., Miller, P. D., Poladian, L. & de Sterke, C. M. Self-similar evolution of self-written waveguides. *Opt. Lett.* **23**, 268–270 (1998).
33. Fermann, M. E., Kruglov, V. I., Thomsen, B. C., Dudley, J. M. & Harvey, J. D. Self-similar propagation and amplification of parabolic pulses in optical fibers. *Phys. Rev. Lett.* **84**, 6010–6013 (2000).
34. Kruglov, V. I., Peacock, A. C., Dudley, J. M. & Harvey, J. D. Self-similar propagation of high-power parabolic pulses in optical fiber amplifiers. *Opt. Lett.* **25**, 1753–1755 (2000).
35. Kruglov, V. I., Peacock, A. C., Harvey, J. D. & Dudley, J. M. Self-similar propagation of parabolic pulses in normal-dispersion fiber amplifiers. *J. Opt. Soc. Am. B* **19**, 461–469 (2002).
36. Boscolo, S., Turitsyn, S. K., Novokshenov, V. Y. & Nijhof, J. H. B. Self-similar parabolic optical solitary waves. *Theor. Math. Phys.* **133**, 1647–1656 (2002).
37. Agrawal, G. P. *Nonlinear Fiber Optics* 4th edn (Academic, Boston, 2007).
38. Hasegawa, A. & Tappert, F. Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion. *Appl. Phys. Lett.* **23**, 142–144 (1973).
39. Mollenauer, L. F., Stolen, R. H. & Gordon, J. P. Experimental observation of picosecond pulse narrowing and solitons in optical fibers. *Phys. Rev. Lett.* **45**, 1095–1098 (1980).
40. Kodama, Y. & Hasegawa, A. Nonlinear pulse propagation in a monomode dielectric guide. *IEEE J. Quant. Electron.* **QE-23**, 510–524 (1987).
41. Dudley, J. M., Genty, G. & Coen, S. Supercontinuum generation in photonic crystal fiber. *Rev. Mod. Phys.* **78**, 1135–1184 (2006).
42. Tomlinson, W. J., Stolen, R. H. & Johnson, A. M. Optical wave breaking of pulses in nonlinear optical fibers. *Opt. Lett.* **10**, 457–459 (1985).
43. Rosenberg, C. J., Anderson, D., Desaix, M., Johannisson, P. & Lisak, M. Evolution of optical pulses towards wave breaking in highly nonlinear fibres. *Opt. Commun.* **273**, 272–277 (2007).
44. Anderson, D., Desaix, M., Karlson, M., Lisak, M. & Quiroga-Teixeiro, M. L. Wave-breaking-free pulses in nonlinear optical fibers. *J. Opt. Soc. Am. B* **10**, 1185–1190 (1993).
45. Tamura, K. & Nakazawa, M. Pulse compression by nonlinear pulse evolution with reduced optical wave breaking in erbium-doped fiber amplifiers. *Opt. Lett.* **21**, 68–70 (1996).
46. Trebino, R. *Frequency-Resolved Optical Gating. The Measurement of Ultrashort Laser Pulses* (Kluwer Academic, Dordrecht, 2002).
47. Kruglov, V. I. & Harvey, J. D. Asymptotically exact parabolic solutions of the generalized nonlinear Schrödinger equation with varying parameters. *J. Opt. Soc. Am. B* **23**, 2541–2550 (2006).
48. Finot, C. Influence of the pumping configuration on the generation of optical similaritons in optical fibers. *Opt. Commun.* **249**, 553–561 (2005).
49. Finot, C., Parmigiani, F., Petropoulos, P. & Richardson, D. J. Parabolic pulse evolution in normally dispersive fiber amplifiers preceding the similariton formation regime. *Opt. Express* **14**, 3161–3170 (2006).
50. Wabnitz, S. Analytical dynamics of parabolic pulses in nonlinear optical fiber amplifiers. *IEEE Photon. Technol. Lett.* **19**, 507–509 (2007).
51. Finot, C., Millot, G., Billet, C. & Dudley, J. M. Experimental generation of parabolic pulses via Raman amplification in optical fiber. *Opt. Express* **11**, 1547–1552 (2003).
52. Peacock, A. C., Broderick, N. G. R. & Monro, T. M. Numerical study of parabolic pulse generation in microstructured fibre Raman amplifiers. *Opt. Commun.* **218**, 167–172 (2003).
53. Ozeki, Y., Takushima, Y., Aiso, K., Taira, K. & Kikuchi, K. Generation of 10 GHz similariton pulse trains from 1.2 km-long erbium-doped fibre amplifier for application to multi-wavelength pulse sources. *Electron. Lett.* **40**, 1103–1104 (2004).
54. Peacock, A. C., Kruhlak, R. J., Harvey, J. D. & Dudley, J. M. Solitary pulse propagation in high gain optical fiber amplifiers with normal group velocity dispersion. *Opt. Commun.* **206**, 171–177 (2002).
55. Chang, G., Galvanauskas, A., Winful, H. G. & Norris, T. B. Dependence of parabolic pulse amplification on stimulated Raman scattering and gain bandwidth. *Opt. Lett.* **29**, 2647–2649 (2004).
56. Soh, D. B. S., Nilsson, J. & Grudinin, A. B. Efficient femtosecond pulse generation using a parabolic amplifier combined with a pulse compressor. I. Stimulated Raman-scattering effects. *J. Opt. Soc. Am. B* **23**, 1–9 (2006).
57. Soh, D. B. S., Nilsson, J. & Grudinin, A. B. Efficient femtosecond pulse generation using a parabolic amplifier combined with a pulse compressor. II. Finite gain-bandwidth effect. *J. Opt. Soc. Am. B* **23**, 10–19 (2006).
58. Finot, C., Pitois, S., Millot, G., Billet, C. & Dudley, J. M. Numerical and experimental study of parabolic pulses generated via Raman amplification in standard optical fibers. *IEEE J. Sel. Top. Quant. Electron.* **10**, 1211–1218 (2004).
59. Zaouter, Y. *et al.* Third-order spectral phase compensation in parabolic pulse compression. *Opt. Express* **15**, 9372–9377 (2007).
60. Finot, C., Millot, G. & Dudley, J. M. Asymptotic characteristics of parabolic similariton pulses in optical fiber amplifiers. *Opt. Lett.* **29**, 2533–2535 (2004).
61. Billet, C., Dudley, J. M., Joly, N. & Knight, J. C. Intermediate asymptotic evolution and photonic bandgap fiber compression of optical similaritons around 1550 nm. *Opt. Express* **13**, 3236–3241 (2005).
62. Maine, P., Strickland, D., Bado, P., Pessot, M. & Mourou, G. Generation of ultrahigh peak power pulses by chirped pulse amplification. *IEEE J. Quant. Electron.* **24**, 398–403 (1988).
63. Limpert, J. *et al.* High-power femtosecond Yb-doped fiber amplifier. *Opt. Express* **10**, 628–638 (2002).
64. Price, J. H. V. *et al.* Soliton transmission and supercontinuum generation in holey fiber, using a diode pumped Ytterbium fiber source. *Opt. Express* **10**, 382–387 (2002).
65. Malinowski, A. *et al.* Ultrashort-pulse Yb^{3+} fiber based laser and amplifier system producing > 25 W average power. *Opt. Lett.* **29**, 2073–2075 (2004).
66. Nicholson, J. W., Yablon, A. D., Westbrook, P. S., Feder, K. S. & Yan, M. F. High power, single mode, all-fiber source of femtosecond pulses at 1550 nm and its use in supercontinuum generation. *Opt. Express* **12**, 3025–3034 (2004).
67. Schreiber, T., Nielsen, C. K., Ortac, B., Limpert, J. & Tünnermann, A. Microjoule-level all-polarization-maintaining femtosecond fiber source. *Opt. Lett.* **31**, 574–576 (2006).
68. Dupriez, P. *et al.* High-power, high repetition rate picosecond and femtosecond sources based on Yb-doped fiber amplification of VECSELs. *Opt. Express* **14**, 9611–9616 (2006).
69. Cregan, R. F. *et al.* Single-mode photonic band gap guidance of light in air. *Science* **285**, 1537–1539 (1999).
70. Kibler, B., Billet, C., Ferriere, R. & Dudley, J. M. All-fiber source of 20 fs pulses at 1550 nm using two stage linear-nonlinear compression of parabolic similaritons. *IEEE Photon. Technol. Lett.* **18**, 1831–1833 (2006).
71. Ilday, F. Ö., Buckley, J. R., Lim, H., Wise, F. W. & Clark, W. G. Generation of 50-fs, 5-nJ pulses at 1.03 μm from a wave-breaking-free fiber laser. *Opt. Lett.* **28**, 1365–1367 (2003).
72. Ilday, F. Ö., Buckley, J. R., Clark, W. G. & Wise, F. W. Self-similar evolution of parabolic pulses in a laser. *Phys. Rev. Lett.* **92**, 213902 (2004).
73. Wise, F. W. Similariton lasers generate high energy femtosecond pulses. *Opt. Photon. News* **15**, 45 (2004).
74. Nielsen, C. K. *et al.* Self-starting self-similar all-polarization maintaining Yb-doped fiber laser. *Opt. Express* **13**, 9346–9351 (2005).
75. Ruehl, A., Hundertmark, H., Wandt, D., Fallnich, C. & Kracht, D. 0.7 W all-fiber erbium oscillator generating 64 fs wave breaking-free pulses. *Opt. Express* **13**, 6305–6309 (2005).
76. Ortac, B., Hideur, A., Chedot, C., Martel, G. & Limpert, J. Self-similar low-noise femtosecond ytterbium-doped double-clad fiber laser. *Appl. Phys. B* **85**, 63–68 (2006).
77. Ilday, F. Ö., Wise, F. W. & Kärtner, F. X. Possibility of self-similar pulse evolution in a Ti-sapphire laser. *Opt. Express* **12**, 2731–2738 (2004).
78. Kalashnikov, V. L. *et al.* Approaching the microjoule frontier with femtosecond laser oscillators: Theory and comparison with experiment. *New J. Phys.* **7**, 217 (2005).
79. Bélanger, P. A. On the profile of pulses generated by fiber lasers: the highly-chirped positive dispersion regime (similariton). *Opt. Express* **14**, 12174–12182 (2006).
80. Ruehl, A. *et al.* Dynamics of parabolic pulses in an ultrafast fiber laser. *Opt. Lett.* **31**, 2734–2736 (2006).
81. Finot, C. & Millot, G. Synthesis of optical pulses by use of similaritons. *Opt. Express* **12**, 5104–5109 (2004).
82. Parmigiani, F. *et al.* Ultra-flat SPM-broadened spectra in a highly nonlinear fiber using parabolic pulses formed in a fiber Bragg grating. *Opt. Express* **14**, 7617–7622 (2006).
83. Finot, C., Pitois, S. & Millot, G. Regenerative 40-Gbit/s wavelength converter based on similariton generation. *Opt. Lett.* **30**, 1776–1778 (2005).
84. Hirooka, T. & Nakazawa, M. Optical adaptive equalization of high-speed signals using time-domain optical Fourier transformation. *J. Lightwave Technol.* **24**, 2530–2540 (2006).
85. Parmigiani, F., Petropoulos, P., Ibsen, M. & Richardson, D. J. Pulse retiming based on XPM using parabolic pulses formed in a fiber Bragg grating. *IEEE Photon. Technol. Lett.* **18**, 829–831 (2006).
86. Hirooka, T. & Nakazawa, M. Parabolic pulse generation by use of a dispersion-decreasing fiber with normal group-velocity dispersion. *Opt. Lett.* **29**, 498–500 (2004).
87. Kibler, B. *et al.* Parabolic pulse generation in comb-like profiled dispersion decreasing fibre. *Electron. Lett.* **42**, 965–966 (2006).
88. Latkin, A. I., Turitsyn, S. K. & Sysoliatin, A. A. Theory of parabolic pulse generation in tapered fiber. *Opt. Lett.* **32**, 331–333 (2007).
89. Kruglov, V. I., Peacock, A. C. & Harvey, J. D. Exact self-similar solutions of the generalized nonlinear Schrödinger equation with distributed coefficients. *Phys. Rev. Lett.* **90**, 113902 (2003).
90. Kruglov, V. I., Mechin, D. & Harvey, J. D. Self-similar solutions of the generalized Schrödinger equation with distributed coefficients. *Opt. Express* **12**, 6198–6207 (2004).
91. Chen, S. & Yi, L. Chirped self-similar solutions of a generalized nonlinear Schrodinger equation model. *Phys. Rev. E* **71**, 016606 (2005).
92. Mechin, D., Im, S. H., Kruglov, V. I. & Harvey, J. D. Experimental demonstration of similariton pulse compression in a comblike dispersion-decreasing fiber amplifier. *Opt. Lett.* **31**, 2106–2108 (2006).
93. Finot, C., Provost, L., Petropoulos, P. & Richardson, D. J. Parabolic pulse generation through passive nonlinear pulse reshaping in a normally dispersive two segment fiber device. *Opt. Express* **15**, 852–864 (2007).
94. Serkin, V. N. & Hasegawa, A. Novel soliton solutions of the nonlinear Schrödinger equation model. *Phys. Rev. Lett.* **85**, 4502–4505 (2000).
95. Serkin, V. N. & Hasegawa, A. Exactly integrable nonlinear Schrodinger equation models with varying dispersion, nonlinearity and gain: Application for soliton dispersion managements. *IEEE J. Sel. Top. Quant. Electron.* **8**, 418–431 (2002).
96. Chang, G., Winful, H. G., Galvanauskas, A. & Norris, T. B. Self-similar parabolic beam generation and propagation. *Phys. Rev. E* **72**, 016609 (2005).
97. Ponomarenko, S. A. & Agrawal, G. P. Do solitonlike self-similar waves exist in nonlinear optical media? *Phys. Rev. Lett.* **97**, 013901 (2006).
98. Ponomarenko, S. A. & Agrawal, G. P. Optical similaritons in nonlinear waveguides. *Opt. Lett.* **32**, 1659–1661 (2007).
99. Drummond, P. D. & Kheruntsyan, K. V. Asymptotic solutions to the Gross-Pitaevskii gain equation: Growth of a Bose–Einstein condensate. *Phys. Rev. A* **63**, 013605 (2001).
100. Keçeli, M., Ilday, F. Ö. & Oktel, M. Ö. Ansatz from nonlinear optics applied to trapped Bose–Einstein condensates. *Phys. Rev. A* **75**, 035601 (2007).

Acknowledgements

Our contributions to this field have benefited from valuable collaborations and discussions with numerous colleagues and friends. We extend thanks to J. D. Harvey, V. I. Kruglov, R. Leonhardt, B. C. Thomsen, A. C. Peacock, M. E. Fermann, J. H. V. Price, N. G. R. Broderick, P. Petropoulos, L. Larger, C. Billet, B. Kibler, J. C. Knight and N. Joly. J.M.D. extends further thanks to M. Segev for sparking his interest in the wider aspects of optical self-similarity.

Correspondence and requests for materials should be addressed to J.M.D.

Competing financial interests

The authors declare no competing financial interests.