Anomalous Dispersion in Er$^{3+}$- and Yb$^{3+}$-Doped Fibers

D. V. Korobkin, Jr., V. A. Semenov, and E. M. Dianov

Abstract—An experimental and theoretical study of anomalous dispersion in Er$^{3+}$- and Er$^{3+}$-Yb$^{3+}$-doped fibers has been developed. Anomalous time delay caused by both absorption and emission at 1.535 μm has been theoretically calculated and experimentally measured. A pump power dependence of anomalous time delay in rare-earth-doped fibers has been theoretically calculated and experimentally investigated. It has been shown that pump power fluctuations lead to propagation time jitter in Er$^{3+}$-doped fiber amplifiers. The pulse interaction due to refractive index change caused by gain saturation is predicted. It has been shown that for Er$^{3+}$-doped fibers with SiO$_2$-GeO$_2$ core composition, the anomalous dispersion per 1-dB gain is twice that of fibers with SiO$_2$-Al$_2$O$_3$ core, which is caused by gain curve form difference. A scheme of mutual compensation of intrinsic fiber dispersion and anomalous dispersion caused by Er$^{3+}$ in the region 1.532–1.537 μm has been suggested.

I. INTRODUCTION

The intimate relation between dispersion and absorption as expressed by the Kramers-Kronig relationships [1] gives rise to anomalous behavior of refractive index in the spectral regions of absorption (emission) in active media such as glasses doped with rare-earth ions. The anomalous behavior of refractive index in extended media such as Er$^{3+}$- and Yb$^{3+}$-doped fibers leads to anomalous time delay and anomalous dispersion, which can be essential for ultrashort pulse amplification [2], [3]. A number of papers considered this problem [4]–[7]. But some important questions, such as the host composition dependence of anomalous dispersion and pump power dependence of anomalous time delay, are still unclarified.

Historically, there is some ambiguity in the definition of the term “anomalous dispersion.” In this paper we refer first-order dispersion $dn/dA$ to as anomalous if $dn/dA > 0$ and normal if $dn/dA < 0$, respectively. $dn/dA > 0$ in the region of strong absorption band. As for second-order (or chromatic, or group velocity) dispersion, we call it negative if $d^2n/dA^2 > 0$ and positive if $d^2n/dA^2 < 0$.

II. EXPERIMENTAL

The spectral dependence of group-time delay in optical fibers can be measured by various techniques [8]–[11]. In order to measure an anomalous dispersion in the region with strong absorption the short (~1 m) pieces of fiber should be used. Corresponding time delays to be measured are considerably small (0.1–0.5 ps) and can be measured only by interferometric technique [12].

The experimental setup is depicted in Fig. 1. Group time delays in rare-earth-doped fibers were measured by fiber Mach-Zehnder interferometer with an air delay line [12]. A galogen lamp/grating monochromator system was used as light source and Ge photodiode as a detector. The signal from detector takes a form of oscillatory pattern (cross-correlation curve). The maximum of this curve corresponds to zero group delay for given wavelength. The spectral width of monochromator slit 5–6 nm was chosen that gave rise to cross-correlation curve width of ~250 μm. The accuracy of curve maximum position determination was 10 μm, which corresponded to 30 fs time delay determination accuracy. Stable interferetion pattern could be observed when rare-earth doped fiber under investigation had up to 10 dB absorption at 1.535 μm, active fiber length being chosen from these reasons.

Two types of fiber was used in the experiments.

Type 1: Er$^{3+}$-doped fiber with SiO$_2$-GeO$_2$ core composition; Er$^{3+}$ ions concentration was 80-mppm, fiber length was 174 cm. In order to excite Er$^{3+}$ ions Tm$^{3+}$ : Al$_2$O$_3$-laser at 0.98 μm was used. The maximum pump power passed through active fiber was $P \approx 22$ mW, the threshold power at 0.98 μm was $P_{th} \approx 0.5$ mW.

Type 2: Er$^{3+}$-Yb$^{3+}$-doped fiber with SiO$_2$-Al$_2$O$_3$ core; Er$^{3+}$ ions concentration was 800 ppm, Yb$^{3+}$–8000 ppm, fiber length was 15.6 cm. The fiber was pumped by Nd$^{3+}$ : YAG laser at $\lambda_p = 1.06$ μm, the maximum pump power passed through active fiber was 70 mW. Threshold power was $P_{th} \approx 50$ mW. The pump depletion in type 2 fiber was negligible, less than 10% at any pumping level and $(\gamma(z) \equiv P(z)/P_{th} \approx 0\text{ const})$.

The spectral dependencies of absorption was measured in a cutback experiment for both types of fibers. The spectral dependence of luminescence was measured in short species being pumped at 0.98 μm (type 1 fiber) and at 0.514 μm (type 2 fiber). All spectral dependencies were digitized. The absorption and emission cross-section data [13] allows us to recalculate the spectral dependence of luminescence to spectral dependence of gain, cm$^{-1}$ for both types of fibers.

III. SPECTRAL DEPENDENCE OF ANOMALOUS TIME DELAY AND ANOMALOUS DISPERSION

Along with experimental determination of time delays we calculated spectral dependencies of refractive index variations
\( \delta n \), anomalous time delay \( \tau \), and anomalous dispersion \( D \) using Kramers-Kronig relationship. The calculated dependencies will be further referred to as theoretical ones.

Fig. 2 shows spectral dependencies of group time delay in type 1 fiber without pumping. Measured data (excluding points in the region 1.5–1.56 \( \mu \text{m} \)) were approximated by fourth-order polynomial. In order to get anomalous time delay caused by \( \text{Er}^{3+} \) ions, the polynomial values were subtracted from data measured. The analogous procedure was carried out for type 2 fiber. All results below are transformed by this manner.

Fig. 3 shows spectral dependencies of anomalous time delay for type 1 fiber 1.74 m long in the region 1.48–1.6 \( \mu \text{m} \), both without pumping and under pumping (pump power 20 mW). It should be noted that theoretical and experimental dependencies are in agreement within the accuracy of the experiment.

As for \( \text{Er}^{3+} \)-Yb\(^{3+}\)-doped fibers, Fig. 4 shows spectral dependence of anomalous time delay per unit length for type 2 fiber without pumping. The theoretical and experimental data are also in good agreement.
Anomalous Time Delay Variations Due to Pump Power Instability

An instability of pump power can give rise to anomalous time delay variation. If $\gamma(z) = \text{const}$ the anomalous time delay will be

$$\tau = \frac{z}{c} \left( \frac{\tau_{\text{ampl}}}{\gamma + 1} + \tau_{\text{abs}} \frac{1}{\gamma + 1} \right).$$

It follows from Fig. 3 that one can adopt for estimations

$$\tau_{\text{ampl}} \approx -\tau_{\text{abs}} = \tau_{\text{anomal}}.$$

The arrival time fluctuations caused by pump power instability will be

$$\Delta \tau = \frac{2\tau_{\text{anomal}} z}{(\gamma + 1)^2} K_{\text{in}} \tau_0,$$

where $K_{\text{in}} = \Delta P/P$-pump power instability. This expression can be transformed using the notion of anomalous time delay in the fiber with $1 \, \text{dB}$ gain $\tau_{1\,\text{dB}}$ [6], [7].

$$\Delta \tau = 2\tau_{1\,\text{dB}} \frac{\alpha L}{(\gamma + 1)^2} K_{\text{in}} \tau_0,$$

where $\tau_{1\,\text{dB}} = 34 \, \text{fs/dB}$ for SiO$_2$-GeO$_2$ host and 23 fs/dB for SiO$_2$-Al$_2$O$_3$ one, $\alpha$-gain, $L$-amplifier length. For a system of $N$ independently pumped amplifiers the corresponding expression is

$$\Delta \tau = 2\tau_{1\,\text{dB}} \frac{\alpha L}{(\gamma + 1)^2} K_{\text{in}} \tau_0 / \sqrt{N},$$

$\alpha L$-here is a total system gain.

Consider a communication system with parameters described in [14]. The anomalous time delay variation for $\gamma \approx 10$ and $K_{\text{in}} \approx 5\%$ is $\Delta \tau \approx 50 \, \text{fs}$. This is 30 times less than the action of Gordon-Haus effect [15] and acoustic interaction among the solitons [16] even with sliding filters [14]. Thus, this effect is negligible for real communication systems.

Let us also consider a fiber amplifier with $G \, \text{dB}$ gain in one arm of an interferometer. Anomalous time delay fluctuations caused by pump power instability should be less than optical cycle. At $K_{\text{in}} \approx 5\%$, and providing $\Delta \tau \ll T \approx 5 \, \text{fs}$ and $G \approx 30 \, \text{dB}$ we have $\gamma \approx 20$, and for $\lambda = 0.98 \, \text{um}$ the corresponding pump power is 10 mW.

In order to diminish anomalous time delay fluctuations caused by pump power instability, one can use the following methods:

1) Decreasing pump instability, $K_{\text{in}}$
2) Increasing pump power.
3) Using operating wavelength with $\tau_{\text{anomal}} = 0$ (see Fig. 3). But this entails a 50% gain decrease for SiO$_2$-GeO$_2$ host $\text{Er}^{3+}$ doped fibers and 20% for SiO$_2$-Al$_2$O$_3$ one.

B. Pulse Interaction

Strong signal can affect inversion population of gain medium due to the effect of gain saturation. If the signal is in the form of pulse pairs, the first pulse having changed inversion changes the effective index of refraction [17]. The
second pulse propagates in the medium with slightly different $\delta n$ and thus slightly different group velocity. This interaction can be significant for pulses that have energy comparable with the energy of saturation $E_s$. For Er$^{3+}$-doped fibers $E_s \approx 10 \text{ J/cm}^2$ [18].

In order to consider this problem in more details one should take into account the relative values of homogeneous and inhomogeneous broadening and the ratio of pulse energy $E_p$ to saturation energy $E_s$. For Er$^{3+}$-doped optical fibers at room temperature the values of inhomogeneous broadening (100 cm$^{-1}$) and homogeneous broadening (50 cm$^{-1}$) are close to each other [19]. Thus for rough estimations one can consider that the gain curve form doesn’t change when gain is saturated. Fig. 3 shows that this interaction takes the form of attraction for gain medium.

For soliton communication systems [14] the corresponding value of pulse energy $E_p \sim 10^{-7} \text{ J/cm}^2, E_p/E_s \sim 10^{-8}$ and the relative delay is $\sim 3 \cdot 10^{-10}$ ps/dB. For communication system length $10^4$ km (2000 dB gain) the relative delay $\tau_r \approx 6 \cdot 10^{-7}$ ps, which is negligible compared to pulse separation. However, this effect can be experimentally observed for powerful pulses of nanosecond duration.

V. GAIN CURVE FORM DEPENDENCE OF ANOMALOUS DISPERSION

The good agreement between experimental and theoretical results allows us to compute the anomalous chromatic dispersion in rare-earth-doped optical fibers.

It follows from the Kramers-Kronig relationship that maximum value of anomalous dispersion essentially depends on gain (absorption) curve form. The ionic susceptibility can be determined as [6]

$$\chi(\lambda) = \frac{1}{2\pi} n_H \lambda_m \alpha_m f(\lambda)$$

(7)

where $n_H$-the host refractive index, $\lambda_m$-wavelength of maximum gain, $\alpha_m$-peak gain per unit length, $f(\lambda)$-spectral shape defined as the complex $\chi(\lambda)$, normalized to the peak value of the imaginary part. The corresponding refractive index variation

$$\delta n = \frac{1}{4\pi} \lambda_m \alpha_m Re f(\lambda)$$

(8)

In order to evaluate the gain curve form dependence of anomalous dispersion, one can choose the Lorentzian with the appropriate spectral width (gain FWHM) $\Delta \lambda$ for the $f(\lambda)$.

$$\delta n = -\frac{1}{2\pi} \lambda_m \alpha_m \frac{(\lambda - \lambda_m)/\Delta \lambda}{1 + 4(\lambda - \lambda_m)^2/\Delta \lambda^2}$$

(9)

The corresponding maximum value of anomalous dispersion $D_m \sim \alpha_m/\Delta \lambda^2$ takes place at $(\lambda - \lambda_m)/\Delta \lambda = \pm 0.207$. For real gain curves the $\Delta \lambda$-dependence of $D_m$ is more complex but the narrower gain curve the larger anomalous dispersion. In order to compare fibers with different dopant levels, one should transform anomalous time delay and anomalous dispersion for fiber with 1 dB full inversion gain, time delay $\tau_1$ in ps/dB measuring in ps/nm of dispersion $D_1$ in ps/nm dB [6], [7]. Fig. 7 shows spectral dependencies of anomalous dispersion for both fiber types (gain FWHM of type 1 fiber was 10 nm, type 2 fiber was 28 nm, full inversion supposed). The magnitude of anomalous dispersion in type 1 fiber is $\sim 2$ times more than in type 2 one.

VI. COMPENSATION OF INTRINSIC FIBER DISPERSION AND ANOMALOUS DISPERSION IN THE REGION 1.535 $\mu m$

Slopes of silica fiber intrinsic dispersion and anomalous dispersion of excited Er$^{3+}$ ions in the region 1.535 $\mu m$ have opposite signs. So, shifting the wavelength of zero dispersion to 1.535 $\mu m$ and choosing the appropriate dopant level of Er$^{3+}$ ions, one can completely eliminate fiber dispersion in the region 1.532-1.537 $\mu m$ (where Er$^{3+}$ doped fiber amplifiers have the maximum gain). Not only the second order dispersion ($k'' = 0$), but also the higher order dispersions are equal zero. This principle is illustrated at Fig. 8, where a spectral dependence of dispersion in a single-mode fiber (SiO$_2$-GeO$_2$ core composition) doped with Er$^{3+}$ ions is depicted. The Er$^{3+}$ ions are considered to be in complete inversion. The corresponding Er$^{3+}$ concentration necessary for that compensation is 0.07 ppm ($\sim 4$ dB/km gain). In SiO$_2$-Al$_2$O$_3$ fibers the corresponding dopant level is 0.1 ppm ($\sim 6.5$ dB/km gain). For real fiber amplifiers with the inversion varying along the fiber such compensation can be realized in the average. The principle mentioned above can be generalized for this case taking into account the pump depletion.

It should be noted that the results mentioned above (except the pulse interaction) are valid in the case of low signal amplification (absorption). If the signal saturates amplification, it should be taken into account in equations describing population and depopulation of upper laser level. The form of (2) would be conserved but the parameter $\gamma$ would depend not only on pump power but also on signal power.

VII. CONCLUSION

Anomalous time delay caused by both absorption and emission in Er$^{3+}$ and Er$^{3+}$-Yb$^{3+}$-doped fibers has been theo-
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1.0 0.5 0.0 1.0
1520 1525 1530 1535 1540 1545 1550 λ, nm

Fig. 8. Spectral dependence of chromatic dispersion in fiber with compensated by virtue of Er3+ ions dispersion at 1.535 μm.

rgetically calculated and experimentally measured. Theoretical and experimental results found to be in agreement within the accuracy of experiment.

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It has been shown that for Er3+-doped fibers with SiO2-GeO2 core composition the anomalous dispersion per 1 dB gain is twice that of fibers with SiO2-Al2O3 core, which is caused by gain curve form difference.

A scheme of mutual compensation of intrinsic fiber dispersion and anomalous dispersion in the region 1.532-1.537 μm caused by Er3+ ions has been suggested. The Er3+ ions concentration which is necessary for such kind of compensation has been calculated for SiO2-GeO2 and SiO2-Al2O3 types of fiber core host.

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REFERENCES


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