USE OF WAVEGUIDE HOLOGRAMS FOR INPUT AND TRANSMISSION OF INFORMATION ON A LIGHT FIELD THROUGH A SINGLE-MODE OPTICAL FIBER

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1. Introduction

Fiber optics links are widely used in modern communication systems [1, 2]. The wide transmission band of such systems [3, 4] together with the possibilities of single-mode optical fibers capable of transmitting optical signals without phase distortions [5, 6] make it possible to pose the problem of input and transmission of information on the distribution of a light field through optical fibers that is specified, e.g., by the complex amplitude transmittance of an information transparency with allowance for its amplitudes and phases [7]. Then the light field can be reconstructed at the output of the system. From this point of view such a device may be considered as an element for developing a three-dimensional TV system.

In this paper we consider a scheme for input and transmission of information on the complex amplitude transmittance of a one-dimensional transparency through a single-mode fiber communication channel with a waveguide hologram used for input [8]. A waveguide hologram as well as any other kind of hologram makes it possible to form a structure of a light beam that matches the optical fiber employed for input, i.e., provides focusing and input of radiation into a fiber without any additional elements. The waveguide nature of the reconstruction process is an advantage of the waveguide hologram in the scheme under consideration, which provides rapid reconstruction and almost equal amplitudes of waves reconstructed from different areas of the hologram when a short reading pulse is used and the diffraction efficiency is relatively low (or increases in the direction of propagation of the reading signal along the hologram). Holograms used for reading information from a transparency can be recorded by external waves and then read in a waveguide regime [9].

2. Optical Setup with a Waveguide Hologram for Introducing Optical Information into a Single-Mode Optical Fiber

A schematic diagram of an optical setup with specially prepared waveguide holograms for input of information on the structure of a one-dimensional light field into a fiber communication line is shown in Fig. 1. In our analysis we assume that the waveguide hologram WGH (shaded in Fig. 1) is located close to the plane \( z = 0 \) and comprises a specially prepared structure oriented along the \( x \) axis. The waveguide hologram characteristics that are of interest for the application under consideration are given below (see Sec. 5 and [9, 10]).

In order to form a light beam of the required structure and transform the coordinate representation of the complex information \( f(x) \) into serial information with passage to time modulation of the field, the waveguide
The hologram is illuminated by a short radiation pulse in the form of a plane wave \( E_p(x, y, z, t) \) propagating along the \( z \) axis. The one-dimensional information to be transmitted is represented in the form of the complex amplitude transmittance \( t(x) \) of the one-dimensional transparency \( Tr \), whose plane is practically aligned with that of the hologram. In certain cases, this information can be recorded on the hologram itself in addition to the structure that forms the radiation to be introduced into the optical fiber. The reconstructed field is obtained at the entrance to the fiber optics line FOL transmitting the information. The input end has the coordinates \( x_{oo}, y_{oo}, z_{oo} \) and is separated from the hologram plane by the distance \( z_{oo} \).

To provide input of radiation modulated in accordance with the transmitted amplitude-phase information with the required spatial resolution into an optical communication line comprising a single-mode fiber, the waveguide hologram should focus radiation from each hologram element onto the fiber input, i.e., each hologram element should produce a converging spherical wave in the reconstruction process. For definiteness, the notation \( \xi, \eta, \zeta \) is taken for the coordinates in the description of the structure of the hologram and the fields in its body. A converging spherical wave in the plane \( z = 0 \) is described by the expression [11]

\[
E_{oo}(\xi, \eta, 0, t) = \frac{e_{oo}(\xi, \eta) \exp(-i\omega t)}{r_{oo}(P, P_{oo})} \exp(-i\omega t) \exp \left[-i\frac{\omega}{c} r_{oo}(P, P_{oo}) \right], \tag{1}
\]

where

\[
r_{oo}(P, P_{oo}) = r_{oo}(\xi, \eta, 0, x_{oo}, y_{oo}, z_{oo})
\]
is the distance between an arbitrary point in the plane $z = 0$ and the point with the coordinates $x_{00}, y_{00}, z_{00}$ that corresponds to wave focusing. In the amplitude factor of this expression we may set

$$r_{00} = \rho_{00} = (x_{00}^2 + y_{00}^2 + z_{00}^2)^{1/2},$$

where $\rho_{00}$ is the distance between the origin of coordinates and the focusing point, i.e., we may assume that the wave amplitude in the plane $z = 0$ is independent of the coordinates $\xi, \eta$. In the case where the reasonable condition

$$\xi, \eta \ll \rho_{00}$$

is satisfied, we may restrict our consideration of the phase factor to the Fresnel approximation \cite{11}:

$$r_0(P, P_{00}) = [((\xi - x_{00})^2 + (\eta - y_{00})^2 + (-z_{00})^2)^{1/2} = \rho_{00} - \frac{\xi x_{00} + \eta y_{00}}{\rho_{00}} + \frac{\xi^2 + \eta^2}{2\rho_{00}}. $$

Taking into account the limited size of the acting field in the plane $z = 0$, which equals the hologram size, we may describe the field near the focus by the Kirchhoff integral \cite{11}

$$E_{00}(x, y, z, t) = \frac{-i}{\lambda \rho_{00}} \int e^{00}(\xi, \eta) \exp(-i\omega t') |_{t' = t - r_0/c} \exp(i2\pi r_0) \int \exp \left( \frac{i\omega}{c} \left( \frac{\xi^2 + \eta^2}{2\rho_{00}} + \frac{\xi x_{00} + \eta y_{00}}{\rho_{00}} \right) \right) \times \exp \left( \frac{i\omega}{c} r_0 \right) d\xi d\eta, $$

(2)

where

$$r_0(P, P_{00}) = r_0(\xi, \eta, 0, x, y, z)$$

is the distance between an arbitrary point in the region selected by a diaphragm in the plane $z = 0$ and the area into which the radiation is focused. The average direction of wave propagation is given by the direction cosines

$$\cos \vartheta_{x_{00}} = \frac{x_{00}}{\rho_{00}}, \quad \cos \vartheta_{y_{00}} = \frac{y_{00}}{\rho_{00}}, \quad \cos \vartheta_{z_{00}} = \frac{z_{00}}{\rho_{00}},$$

which satisfy the trigonometric condition

$$\cos \vartheta_{x_{00}}^2 + \cos \vartheta_{y_{00}}^2 + \cos \vartheta_{z_{00}}^2 = 1.$$  

The given expression describes the field representing a converging spherical wave having a singularity at the point with the coordinates $x_{00}, y_{00}, z_{00}$ and takes into account diffraction distortions caused by the limiting diaphragm.

A schematic diagram of a setup for recording a waveguide hologram with external light waves is shown in Fig. 2. The hologram should comprise the required focusing structure along the $\xi$ and $\eta$ axes. The hologram
may exhibit three-dimensional effects along the $\zeta$ axis. Allowance for such effects in the hologram may arise from a desired level of resolution. In actuality, a resolved spatial element of size $2\Delta \xi \sim 0.1$ mm (see Fig. 1) should provide focusing of the wave reconstructed from it onto the input end of the fiber communication line, i.e., this element should comprise, when recording the hologram, a rather large number of interference maxima. Hence, the hologram should be recorded using a high carrier spatial frequency and a structure period of the order of the wavelength $\lambda \sim 1$ $\mu$m. Consequently, even in the case of a small thickness $2\zeta_0 \sim 5$ $\mu$m of the waveguide hologram, three-dimensional effects may be of considerable importance in certain geometries of the optical scheme. The size of $2\xi_0 \times 2\eta_0 \times 2\zeta_0$ (see Fig. 1) with the conditions

$$2\xi_0 \gg 2\eta_0 \gg 2\zeta_0$$

satisfied, will be taken for the hologram operating region.

The hologram is recorded by using sources of cw radiation of frequency $\omega_1$. A spherical signal wave $E_0(x, y, z, t)$ in the body of the medium where the hologram is recorded is described in a way similar to (1):

$$E_0(\xi, \eta, \zeta, t) = \varepsilon_0(\xi, \eta, \zeta) \exp(-i\omega_1 t) \frac{\varepsilon_0(\zeta)}{\rho_0(P_0, P)} \exp(-i\omega_1 t) \exp \left[ \pm \frac{i\omega_1}{c} r_0(P_0, P) \right],$$

where

$$\rho_0 = (x_0^2 + y_0^2 + z_0^2)^{1/2}$$

is the distance between the wave source (the point $P_0$ with the coordinates $x_0, y_0, z_0$) and the origin of coordinates;

$$r_0(P_0, P) = r_0(x_0, y_0, z_0, \xi, \eta, \zeta)$$

is the distance between the wave source and an arbitrary point $P$ with the coordinates $\xi, \eta, \zeta$ in the body of a recording element. The plus sign in the phase factor corresponds to a diverging wave, whereas the minus
sign corresponds to a converging one. If the natural condition

\[ 2\zeta_0 \ll r_0(P_0, P) \]

is satisfied, then expression (3) can be simplified. In the analysis of amplitude effects, the hologram may be considered thin for a relatively low beam divergence. Consequently, we may neglect the dependence of the amplitude factor on \( \zeta \), i.e., we may set

\[ \varepsilon_0(\zeta) = \varepsilon_0. \]

In the phase factor

\[ \exp \left[ \pm i \frac{\omega_1}{c} r_0(P_0, P) \right] \]

we may restrict our consideration to an approximate expression that is similar to the Fresnel approximation to a certain extent, i.e., we may take

\[
\begin{align*}
  r_0(P_0, P) &= \left[ (\xi - x_0)^2 + (\eta - y_0)^2 + (\zeta - z_0)^2 \right]^{1/2} \\
  &= \left[ \rho_0^2 - 2(\xi x_0 + \eta y_0 + \zeta z_0) + (\xi^2 + \eta^2 + \zeta^2) \right]^{1/2} \\
  &\approx \frac{\rho_0 - \xi x_0 + \eta y_0 + \zeta z_0}{\rho_0} + \frac{\xi^2 + \eta^2 + \zeta^2}{2\rho_0}.
\end{align*}
\]

(4)

The average direction of wave propagation can be described by the direction cosines

\[
\begin{align*}
  \cos \vartheta_{x_0} &= \frac{x_0}{\rho_0}, \\
  \cos \vartheta_{y_0} &= \frac{y_0}{\rho_0}, \\
  \cos \vartheta_{z_0} &= \frac{z_0}{\rho_0},
\end{align*}
\]

(5)

satisfying the standard trigonometric constraint

\[ \cos \vartheta_{x_0}^2 + \cos \vartheta_{y_0}^2 + \cos \vartheta_{z_0}^2 = 1. \]

Finally, the spherical signal wave in the body of a recording element is described similarly to (2) by the expression [see (3)-(5)]

\[
E_0(\xi, \eta, \zeta, t) = \frac{\varepsilon_0}{\rho_0} \exp(-i\omega_1 t) \exp \left\{ \pm i \frac{\omega_1}{c} \left[ \rho_0 - (\xi \cos \vartheta_{x_0} + \eta \cos \vartheta_{y_0} + \zeta \cos \vartheta_{z_0}) + \frac{\xi^2 + \eta^2 + \zeta^2}{2\rho_0} \right] \right\}.
\]

(6)

When a hologram is recorded, the reference wave \( E_r(x, y, z, t) \) enters the recording region. For the sake of uniform representation of the fields, we assume that the wave is produced by a point source, which is at the distance

\[ \rho_r = (x_r^2 + y_r^2 + z_r^2)^{1/2} \]

from the coordinate origin, i.e., a diverging spherical wave is assumed. If the actual wave is plane, we may consider this case as a point source located at infinity, \( \rho_r \to \infty \). The reference wave near the recording element region is represented similarly to (1) and (3) by

\[
E_r(\xi, \eta, \zeta, t) = \frac{\varepsilon_r(\zeta)}{r_r(P_r, P)} \exp(-i\omega_1 t) \exp \left[ i \frac{\omega_1}{c} r_r(P_r, P) \right],
\]

59
where

\[ r_r(P_r, P) = r_r(x_r, y_r, z_r, \xi, \eta, \zeta) \]

is the distance between the wave source located at the point \( P_r \) with the coordinates \( x_r, y_r, z_r \) and an arbitrary point \( P \) with the coordinates \( \xi, \eta, \zeta \) in the body of a recording element. Similarly to the case of the signal wave [see (4), (5)] we obtain

\[ r_r(P_r, P) \approx \rho_r - \frac{\xi x_r + \eta y_r + \zeta z_r}{\rho_r} + \frac{\xi^2 + \eta^2 + \zeta^2}{2\rho_r} \]

and

\[ \cos \vartheta_{xr} = \frac{x_r}{\rho_r}, \]
\[ \cos \vartheta_{yr} = \frac{y_r}{\rho_r}, \]
\[ \cos \vartheta_{zr} = \frac{z_r}{\rho_r}, \]

under the trigonometric constraint

\[ \cos \vartheta_{xr} + \cos \vartheta_{yr} + \cos \vartheta_{zr} = 1. \]

Thus, the reference wave can be represented in a form similar to (6):

\[ E_r(\xi, \eta, \zeta, t) = \frac{e_r}{\rho_r} \exp(-i\omega_1 t) \exp \left\{ i\frac{\omega_1}{c} \left[ \rho_r - (\xi \cos \vartheta_{xr} + \eta \cos \vartheta_{yr} + \zeta \cos \vartheta_{zr}) + \frac{\xi^2 + \eta^2 + \zeta^2}{2\rho_r} \right] \right\}. \quad (7) \]

As a result, the fields \( E_0(\xi, \eta, \zeta, t) \) [see (6)] and \( E_r(\xi, \eta, \zeta, t) \) [see (7)] form the intensity distribution

\[ I(\xi, \eta, \zeta) = \left( \frac{e_r}{\rho_r} \right)^2 + \frac{\varepsilon_0 e_r}{\rho_0 \rho_r} \exp \left\{ i\frac{\omega_1}{c} \left[ (\pm \rho_0 - \rho_r) + i\frac{\omega_1}{c} \left[ \pm \xi^2 + \eta^2 + \zeta^2 \frac{2\rho_0}{2\rho_r} - \frac{\xi^2 + \eta^2 + \zeta^2}{2\rho_r} \right] \right] \right\} + \text{c.c.} \]
\[ = \rho_r + I_2 + \Delta I(\xi, \eta, \zeta). \quad (8) \]

We note again that the plus sign corresponds to a diverging wave whereas the minus sign corresponds to a converging one. The fraction of the intensity that depends on the coordinates can be represented as a sum of two components:

\[ I(\xi, \eta, \zeta) = I_1 + I_2 + I_3(\xi, \eta, \zeta) + I_4(\xi, \eta, \zeta). \]

Under ordinary conditions of holographic recording [12], the recorded information is related linearly to the coordinate-dependent fraction of the intensity distribution \( \Delta I(\xi, \eta, \zeta) \). The resulting hologram will be characterized by the distribution of the permittivity perturbation \( \Delta \varepsilon(\xi, \eta, \zeta) \). The permittivity distribution recorded in a recording element with (8) taken into account is given by the expression

\[ \varepsilon(\xi, \eta, \zeta) = \varepsilon + \Delta \varepsilon_0 + \Delta \varepsilon(\xi, \eta, \zeta) = \varepsilon + \Delta \varepsilon_0 + g \Delta I(\xi, \eta, \zeta) \]
\[ = \varepsilon + \Delta \varepsilon_1 + \Delta \varepsilon_2 + \Delta \varepsilon_3(\xi, \eta, \zeta) + \Delta \varepsilon_4(\xi, \eta, \zeta), \quad (9) \]
where [see (8)]

\[
\Delta \varepsilon_0 = \Delta \varepsilon_1 + \Delta \varepsilon_2 = g(I_1 + I_2),
\]

\[
\Delta \varepsilon_3(\xi, \eta, \zeta) = gI_3(\xi, \eta, \zeta),
\]

\[
\Delta \varepsilon_4(\xi, \eta, \zeta) = gI_4(\xi, \eta, \zeta).
\]

The constant component of the hologram permittivity \(\varepsilon_0\) comprises the permittivity \(\varepsilon\) characterizing the unperturbed recording medium and the constant additive \(\Delta \varepsilon_0\) caused by the action of the constant components of the intensity:

\[
\varepsilon_0 = \varepsilon + \Delta \varepsilon_1 + \Delta \varepsilon_2 = \varepsilon + g(I_1 + I_2).
\]

Let us consider the process of reading the phase-amplitude information by a short pulse (of duration \(\tau\)) of a reconstructing wave. Let us assume that the spatial resolution of the information to be read is \(2\Delta \xi\). When the hologram is reconstructed, a resolved element of size \(2\Delta \xi\) should form a converging spherical wave focused onto the waveguide entrance (see Fig. 1). The emission of this wave from the hologram output passes through a transparency with a complex amplitude transmittance \(t(\xi)\), which forms phase and amplitude modulation of the emission in accordance with the information to be transmitted. When reading information, the pulse duration must match the desired resolution. For a spatial resolution of \(2\Delta \xi\), the pulse duration \(\tau\) is determined by the relationship

\[
\tau = \frac{2\Delta \xi}{v_\xi},
\]

where \(v_\xi\) is the speed of optical-signal propagation in the hologram plane along the \(\xi\) axis. An area of the hologram of dimension \(2\Delta \xi\) with

\[\xi' - \Delta \xi \leq \xi \leq \xi' + \Delta \xi,\]

from which the signal is read, moves at the speed \(v_\xi\), where

\[\xi' = v_\xi t.\]

The reconstructing wave of frequency \(\omega_2 \neq \omega_1\) is also taken to be spherical and to be produced by a point light source separated from the origin of coordinates by a distance

\[
\rho_p = (x_p^2 + y_p^2 + z_p^2)^{1/2}.
\]

For sequential reading, as noted above, the wave is actually a pulse of duration \(\tau\). The reconstructing wave in the area of a recording element has a form similar to (1) and (3):

\[
E_p(\xi, \eta, \zeta, t) = \frac{\varepsilon_p(\xi, \eta, \zeta, t)}{r_p(P_p, P)} \exp(-i\omega_2 t) \exp \left[ \frac{i\omega_2}{c} r_p(P_p, P) \right].
\]

In this expression, the temporal behavior of the amplitude \(\varepsilon_p(\xi, \eta, \zeta, t)\) describes the pulse structure, and

\[r_p(P_p, P) = r_p(x_p, y_p, z_p, \xi, \eta, \zeta)\]

is the distance between the wave source located at the point \(P_p\) with the coordinates \(x_p, y_p, z_p\) and an arbitrary point \(P\) with the coordinates \(\xi, \eta, \zeta\) in the body of a recording element. Similarly to the case of the signal wave [see (4)-(6)] we obtain

\[
E_p(\xi, \eta, \zeta, t) = \frac{\varepsilon_p(t)}{\rho_p} \exp(-i\omega_2 t) \exp \left\{ \frac{i\omega_2}{c} \rho_p - (\xi \cos \vartheta_{xp} + \eta \cos \vartheta_{yp} + \zeta \cos \vartheta_{zp}) + \frac{\xi^2 + \eta^2 + \zeta^2}{2\rho_p} \right\},
\]

(10)
where
\[
\begin{align*}
\cos \theta_{xp} &= \frac{x_p}{p_p}, \\
\cos \theta_{yp} &= \frac{y_p}{p_p}, \\
\cos \theta_{zp} &= \frac{z_p}{p_p}
\end{align*}
\]
are the direction cosines, which satisfy the trigonometric condition
\[
\cos \theta_{xp}^2 + \cos \theta_{yp}^2 + \cos \theta_{zp}^2 = 1.
\]

To make the physical principles clear, let us analyze separately each component of the reconstructed field neglecting space effects in the hologram, i.e., we assume that \( \zeta = 0 \). Then on the basis of (8) and (9) we obtain
\[
\Delta \varepsilon_3(\xi, \eta, 0) = \frac{g \varepsilon_0 \varepsilon_r}{\rho_0 \rho_r} \exp \left\{ i \frac{\omega_1}{c} \left( \pm \rho_0 - \rho_r \right) + i \frac{\omega_1}{c} \left( \frac{\xi^2 + \eta^2 - \xi^2 + \eta^2}{2 \rho_0 \rho_r} \right) - i \frac{\omega_1}{c} \left[ \xi (\pm \cos \theta_x - \cos \theta_r) + \eta (\pm \cos \theta_y - \cos \theta_r) \right] \right\}. \tag{11}
\]

With the complex amplitude transmittance \( t(\xi) \) of the transparency taken into account, the field at the fiber input is defined by a Kirchhoff integral in the quasi-static approximation [11]:
\[
E_3(x, y, z, t) = -i \frac{1}{\lambda_2 \rho_3} \int E_\rho(\xi, \eta, 0, t') t(\xi) \Delta \varepsilon_3(\xi, \eta, 0) \big|_{t' = -r_3/c} d\xi d\eta,
\]
where
\[
r_3(P, P_3) = r_3(\xi, \eta, 0, x, y, z)
\]
is the distance between an arbitrary point \( P \) of the hologram in the plane \( z = 0 \) with the coordinates \( \xi, \eta, 0 \) and a point \( P_3 \) with the coordinates \( x, y, z \) in the observation zone, and
\[
\rho_3 = (x_3^2 + y_3^2 + z_3^2)^{1/2}
\]
is the distance between the origin of coordinates and the point of observation. With (10) and (11) taken into consideration, this yields
\[
E_3(x, y, z, t) = -i \frac{g \varepsilon_0 \varepsilon_r}{\lambda_2 \rho_0 \rho_r \rho_3} \exp \left\{ -i \frac{\omega_2 \xi}{c} \left( \pm \rho_0 - \rho_r \right) + i \frac{\omega_1}{c} \left( \xi \right) \exp \left\{ i \frac{\omega_2}{\omega_1} \left( \xi^2 + \eta^2 \right) \left[ \frac{1}{\rho_0} + \frac{1}{\rho_1} \left( \frac{\xi}{\omega_2} \right) \right] \\
- i \frac{\omega_1}{c} \left[ \frac{\xi \rho_0 + \omega_2 (\pm \cos \theta_x - \cos \theta_r) + \eta (\pm \cos \theta_y - \cos \theta_r) \right] \\
- i \frac{\omega_1}{c} \left[ \frac{\xi \rho_0 + \omega_2 (\pm \cos \theta_x - \cos \theta_r) + \eta (\pm \cos \theta_y - \cos \theta_r) \right] \exp \left[ i \frac{\omega_2}{c} \frac{r_3(P, P_3)}{r_3(P, P_3)} \right] \right\} d\xi d\eta. \tag{12}
\]

Let us compare this expression with (2), which describes a converging spherical wave near the focus with allowance for the bounded wavefront. One can see that the wave \( E_3 \) yields a diverging spherical wave if
the conditions obtained by correlating the coefficients of the quadratic and linear terms in the integrand are satisfied:

\[
\frac{1}{\rho_3} = -\frac{1}{\rho_0} - \frac{\omega_1}{\omega_2} \left( \pm \frac{1}{\rho_0} - \frac{1}{\rho_r} \right),
\]
\[
\cos \vartheta_{x3} = -\cos \vartheta_{xp} - \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{x0} - \cos \vartheta_{xr}),
\]
\[
\cos \vartheta_{y3} = -\cos \vartheta_{yp} - \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{y0} - \cos \vartheta_{yr}).
\] (13)

They describe the direction cosines and the distance between the observation point and the origin of coordinates. Under these conditions, the wave is focused into a point with the coordinates

\[
x_3 = \rho_3 \cos \vartheta_{x3},
\]
\[
y_3 = \rho_3 \cos \vartheta_{y3},
\]
\[
z_3 = (\rho_3^2 + x_3^2 + y_3^2)^{1/2}
\]
\[
= \rho_3 (1 - \cos^2 \vartheta_{x3} - \cos^2 \vartheta_{y3})^{1/2}.
\]

Similarly, the component \(\Delta \varepsilon_4(\xi, \eta, 0)\) of the permittivity perturbation is determined by the expression

\[
\Delta \varepsilon_4(\xi, \eta, 0) = \frac{g_\varepsilon \varepsilon_r}{\rho_0 \rho_r} \exp \left\{ i \frac{\omega_1}{c} (\pm \rho_0 + \rho_r) + i \frac{\omega_1}{2} \left[ \frac{\xi^2 + \eta^2}{2 \rho_0} + \frac{\xi^2 + \eta^2}{2 \rho_r} \right]
\right.
\]
\[
- \frac{i \omega_1}{c} \left[ \eta (\mp \cos \vartheta_{x0} + \cos \vartheta_{xr}) + \xi (\mp \cos \vartheta_{y0} + \cos \vartheta_{yr}) \right] \}
\]

and

\[
E_4(x, y, z, t) = -i \frac{\omega_1}{\lambda_2 \rho_4} \int E_p(\xi, \eta, 0, t') t(\xi) \Delta \varepsilon_4(\xi, \eta, 0) |_{t' = t - r_4/c} \, d\xi \, d\eta
\]
\[
= -i \frac{g_\varepsilon \varepsilon_r}{\lambda_2 \rho_0 \rho_r \rho_4} \exp \left\{ -i \omega_2 t + \frac{i \omega_2}{c} \left[ \rho_p - \frac{\omega_1}{\omega_2} (\pm \rho_0 - \rho_r) \right] \}
\right.
\]
\[
\times \int \varepsilon_p \left[ - \frac{r_4(P, P_4)}{c} \right] t(\xi) \exp \left\{ i \frac{\omega_2}{2c} (\xi^2 + \eta^2) \left[ \frac{1}{\rho_p} - \frac{\omega_1}{\omega_2} (\pm \rho_0 - \rho_r) \right]
\right.
\]
\[
- \frac{i \omega_2}{c} \xi \left[ \cos \vartheta_{xp} - \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{x0} - \cos \vartheta_{xr}) \right]
\]
\[
- \frac{i \omega_2}{c} \eta \left[ \cos \vartheta_{yp} - \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{y0} - \cos \vartheta_{yr}) \right] \}
\exp \left\{ i \frac{\omega_2}{c} r_4(P, P_4) \right\} \, d\xi \, d\eta,
\] (14)

where

\[ r_4(P, P_4) = r_4(\xi, \eta, 0, x, y, z) \]

is the distance from an arbitrary point \(P\) of the hologram in the plane \(z = 0\) with the coordinates \(\xi, \eta, 0\) to a point \(P_4\) with the coordinates \(x, y, z\) in the observation zone, and

\[ \rho_4 = (x_4^2 + y_4^2 + z_4^2)^{1/2} \]

is the separation between the origin of coordinates and the observation point. In the case where the wave \(E_4(x, y, z, t)\) describes a converging spherical wave [see (13)], the following conditions must be satisfied:
The wave is focused into a point with the coordinates

\[ \begin{align*}
    x_4 &= \rho_4 \cos \vartheta_{x4}, \\
    y_4 &= \rho_4 \cos \vartheta_{y4}, \\
    z_4 &= \left( \rho_4^2 - x_4^2 - y_4^2 \right)^{1/2} = \rho_4 \left( 1 - \cos^2 \vartheta_{x4} - \cos^2 \vartheta_{y4} \right)^{1/2}.
\end{align*} \]

Constraints (13) and (15) allow one to find a geometry of optical systems for recording and reconstructing a hologram that provides focusing of radiation onto the entrance of a fiber link.

The displacement of a reading pulse is characterized in expressions (12) and (14) obtained for the fields by the factor

\[ \varepsilon_p \left( t - \frac{\tau_3(P, P_3)}{c} \right) \] or \[ \varepsilon_p \left( t - \frac{\tau_4(P, P_4)}{c} \right). \]

For a reading pulse of duration \( \tau \), the spatial resolution is

\[ 2\Delta x = \tau v_x. \]

To provide the required spatial resolution, the complex amplitude transmittance \( t(\xi) \) of an information transparency should not vary noticeably over the interval \( 2\Delta \xi \). In this case, we can factor \( t(\xi) \) outside the integral sign.

In expressions (12) and (14), we can take the spectrum of the reading signal into account by employing the Fourier integral representation

\[ \varepsilon_p(t) = \int \varepsilon_p(\omega) \exp(-i\omega t) \, d\omega. \] (16)

Let us restrict our considerations to an analysis of expression (12) (in the case of (14) it will suffice to change the corresponding signs). To make the calculations shorter we will use the notation corresponding to (13). The spectral representation allows us to make the following transformation of (12):

\[ \int \left\{ \int \varepsilon_p(\omega) \exp \left[ -i\omega \left( t - \frac{\tau_3(P, P_3)}{c} \right) \right] \, d\omega \right\} \times \exp \left[ -i\frac{\omega_2}{2c} (\xi^2 + \eta^2) \frac{1}{\rho_3} + i\frac{\omega_2}{c} \xi \cos \vartheta_{x3} + i\eta \cos \vartheta_{y3} \frac{\omega_2}{c} \right] \exp \left[ i\frac{\omega_2}{c} \tau_3(P, P_3) \right] \\
= \int \varepsilon_p(\omega) \exp \left( -i\omega t \right) \times \left\{ \int \exp \left[ -i\frac{\omega_2}{2c} (\xi^2 + \eta^2) \frac{1}{\rho_3} + i\frac{\omega_2}{c} \xi \cos \vartheta_{x3} + i\frac{\omega_2}{c} \eta \cos \vartheta_{y3} \right] \exp \left[ i\frac{\omega_2 + \omega}{c} \tau_3(P, P_3) \right] \, d\xi \, d\eta \right\} \, d\omega. \] (17)
This means that each spectral component of the reading signal is focused into a point whose displacement is proportional to $\omega/\omega_2$, i.e., the diffraction structure of the hologram forms the spectrum of the reading pulse near the focus. This fact should be taken into account when feeding radiation into an optical fiber.

In the simplest case, where the spectral structure of the reading signal can be neglected, the finiteness of the pulse can be allowed for in the limits of integration along the $\xi$ axis, which are of importance for the problem under consideration, by taking them equal to $v_x t \pm \Delta \xi$. This corresponds to a reading pulse approximated by a rectangle with a base equal to the resolution

$$2\Delta \xi = \tau v_x.$$ In this case, with accuracy up to unimportant constant factors, by employing for brevity the notation of (13) and (15), we obtain [see (12) and (14)]

$$E_3(x, y, z, t) = t(v_x t) \exp \left( -i\omega_2 t \right) \int_{v_x t - \Delta \xi - \eta_0}^{v_x t + \Delta \xi - \eta_0} \exp \left[ -i\frac{\omega_2}{2c} (\xi^2 + \eta^2) \frac{1}{\rho_3} + i\frac{\omega_2}{c} \xi \cos \vartheta_{3z} + i\omega_2 c \eta \cos \vartheta_{3\eta} \right]$$

$$\times \exp \left[ i\frac{\omega_2}{c} r_3(P, P_3) \right] d\xi d\eta$$

(18)

and

$$E_4(x, y, z, t) = t(v_x t) \exp \left( -i\omega_2 t \right) \int_{v_x t - \Delta \xi - \eta_0}^{v_x t + \Delta \xi - \eta_0} \exp \left[ -i\frac{\omega_2}{2c} (\xi^2 + \eta^2) \frac{1}{\rho_4} + i\frac{\omega_2}{c} \xi \cos \vartheta_{4z} + i\omega_2 c \eta \cos \vartheta_{4\eta} \right]$$

$$\times \exp \left[ i\frac{\omega_2}{c} r_4(P, P_4) \right] d\xi d\eta.$$ (19)

Thus, it is seen that the amplitudes of the reconstructed fields $E_3$ and $E_4$ in the observation zone are determined by the complex amplitude transmittance $t(v_x t)$ of the information transparency. The spatial amplitude and phase modulation of the field is converted into the time modulation for sequential transfer through a fiber communication line.

3. Reconstruction of a Waveguide Hologram with Allowance for Its Spatial Structure

The reconstruction of a waveguide hologram with allowance for its spatial structure can be described quantitatively using several approaches (see, e.g., [13]) that are suitable for analyzing various features of the process. The general approach to the analysis of holograms recorded in three-dimensional media with allowance for diffraction by its spatial structure is based on solving the wave equation

$$\nabla^2 E(x, y, z, t) - \frac{\varepsilon_0 + \Delta \varepsilon(x, y, z)}{c^2} \frac{\partial^2 E(x, y, z, t)}{\partial t^2} = 0$$

(20)

taking into account the permittivity perturbation $\Delta \varepsilon(x, y, z)$ formed in the body of the hologram in the reconstruction process [see (9)]. In the given equation, $\varepsilon_0$ is the coordinate-independent component of the permittivity of the recording material. The boundary conditions for the field $E(x, y, z, t)$ are given by the Sommerfeld conditions [14]. Their principal meaning is that a field at infinity behaves like a spherical wave.
Quantitative relationships for the energy characteristics of the process, the absolute values of the amplitude distributions, and the diffraction efficiencies can be found on the basis of the coupled wave theory [15] elaborated in detail for the case of plane waves. However, there are certain difficulties within the framework of this theory arising in the analysis of the behavior of complex light fields with a complicated structure of the permittivity perturbation in the body of the hologram.

The problem of the relative distribution of the reconstructed field for virtually arbitrary structures of the permittivity perturbation \( \Delta \varepsilon(\xi, \eta, \zeta) \) and the reconstructing wave \( E_p(x, y, z, t) \) and possible nonstationarity of the latter can be solved within the framework of a reasonable, under the given conditions, approximation of small variations of the amplitude of the reconstructing wave propagating across the hologram body (low diffraction efficiency). It will be solved on the basis of Kirchhoff diffraction theory [11] in the quasi-static approximation [16, 17]. Within the framework of this approach, the field determined by Eq. (20) is the sum of the reconstructing wave \( E_p(x, y, z, t) \) [see (10)] and the field perturbation \( \Delta E(x, y, z, t) \):

\[
E(x, y, z, t) = E_p(x, y, z, t) + \Delta E(x, y, z, t).
\]  

This perturbation represents the reconstructed field and is of practical interest for the scheme under consideration. Taking into account the components \( E_p(x, y, z, t), \Delta E(x, y, z, t) \) [see (21)] and \( \varepsilon_0, \Delta \varepsilon(\xi, \eta, \zeta) \) [see (9)], we can reduce Eq. (20) to the form

\[
\nabla^2 E_p + \frac{\varepsilon_0}{c^2} \frac{\partial^2 E_p}{\partial t^2} - \frac{\varepsilon_0 \partial^2(\Delta E)}{c^2} - \frac{\Delta \varepsilon \partial^2 E_p}{c^2} - \frac{\Delta \varepsilon \partial^2(\Delta E)}{c^2} = 0.
\]

For brevity, the arguments are omitted in this expression. We note that the wave \( E_p(x, y, z, t) \) satisfies the wave equation with the coordinate-independent permittivity \( \varepsilon_0 \):

\[
\nabla^2 E_p(x, y, z, t) - \frac{\varepsilon_0}{c^2} \frac{\partial^2 E_p(x, y, z, t)}{\partial t^2} = 0,
\]

and we neglect the quantity of second order of smallness \( \frac{\Delta \varepsilon \partial^2(\Delta E)}{c^2} \), which includes the small perturbations of the permittivity \( \Delta \varepsilon \) and the field \( \Delta E \). This changes the equation for the reconstructed field to the form

\[
\nabla^2 (\Delta E(x, y, z, t)) - \frac{\varepsilon_0}{c^2} \frac{\partial^2 (\Delta E(x, y, z, t))}{\partial t^2} = \frac{\Delta \varepsilon(x, y, z) \partial^2 E_p(x, y, z, t)}{c^2}.
\]

The solution of equation giving the field at a certain observation point with the coordinates \( x, y, z \) at a distance \( \rho \) from the origin of coordinates is given by the Kirchhoff volume integral in the quasi-static approximation [11, 16, 17]

\[
\Delta E(x, y, z, t) = \frac{1}{4\pi \rho c^2} \int \Delta \varepsilon(x, y, z) \frac{\partial^2 E_p(x, y, z, t)}{\partial t^2} \bigg|_{t'=t-r/c} dV.
\]  

The reconstructing wave may be considered quasi-monochromatic, i.e.,

\[
\Delta \omega_2 \ll \omega_2.
\]

As a result, it is described by the expression

\[
\frac{\partial^2 E_p(x, y, z, t)}{\partial t^2} \approx -\omega_2^2 \varepsilon_p(t) \exp(-i\omega_2 t) \exp \left[ \frac{\omega_2}{c} \left( \xi \cos \vartheta_x + \eta \cos \vartheta_y + \zeta \cos \vartheta_z \right) \right] = -\omega_2^2 E_p(x, y, z, t),
\]
where it is taken into account that 
\[ \frac{\partial^2 \xi_p(t)}{\partial t^2}, \quad \frac{\partial \xi_p(t)}{\partial t} \ll \omega_2. \]

Finally, the solution for the reconstructed field of the form (22) is described by the formula [11, 13]

\[ \Delta E(x, y, z, t) = \frac{\omega_2^2}{4\pi \rho c^2} \int \Delta \epsilon(x, y, z) E_p(x, y, z, t') \bigg|_{t' = t - r/c} dV, \]  

where 
\[ \rho = (x^2 + y^2 + z^2)^{1/2} \]

is the distance from the origin of coordinates to the observation point of the reconstructed field with the coordinates \(x, y, z\). The integral is taken over the whole body of the hologram. Nonstationarities of the field are taken into account in the quasi-static approximation, i.e., for slow variations of field amplitudes in time [16, 17]. We emphasize that the solution does not provide proper absolute values of reconstructed waves because it satisfies the equation at small transformation coefficients.

Let us consider the individual components of the reconstructed field. The component (Fig. 3) due to the term \(\Delta \epsilon_3(\xi, \eta, \zeta)\) of the permittivity perturbation at a point with the coordinates \(x, y, z\) separated by a distance \(\rho_3 = (x^2 + y^2 + z^2)^{1/2}\) from the origin of coordinates is given by the expression [see (8), (9), and (10)]

\[ E_3(x, y, z, t) = \frac{g \omega_2^2 \epsilon_0 c_r}{4\pi c^2 \rho_3} \int \varphi_p \left[ t - \frac{r_3(P, P_3)}{c} \right] \xi(\xi) \exp \left\{ i \frac{\omega_2}{c} \left[ \rho_p + \frac{\omega_1}{\omega_2} (\pm \rho_0 - \rho_r) \right] \right\} \]

\[ + \frac{\omega_2}{2c} (\xi^2 + \eta^2 + \zeta^2) \left[ \frac{1}{\rho_p} + \frac{\omega_1}{\omega_2} \left( 1 + \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right] - i \frac{\omega_2}{c} \left[ \cos \vartheta_{xp} + \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{z0} - \cos \vartheta_{zr}) \right] \]

\[ - i \frac{\omega_2}{c} \left[ \cos \vartheta_{xp} + \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{z0} - \cos \vartheta_{zr}) \right] \times \exp \left\{ -i \frac{\omega_2}{c} \left[ t - \frac{r_3(P, P_3)}{c} \right] \right\} dV. \]

In the amplitude factor, the distances may be measured from the plane \(z = 0\), i.e.,

\[ r_3(P, P_3) \simeq r_3(P_{z=0}, P_3) \]

\[ = \left[ (\xi - x)^2 + (\eta - y)^2 + (z - z_0)^2 \right]^{1/2}. \]

Let us separate the terms dependent on \(\zeta\) [see (4)] in the phase factor specified by the frequency \(\omega_2\):

\[ r_3(P, P_3) \simeq r_3(P_{z=0}, P_3) + \frac{\zeta^2}{2\rho_3} - \zeta \cos \vartheta_{z3}. \]

Here,

\[ \rho_3 = (x_3^2 + y_3^2 + z_3^2)^{1/2} \]

is the distance between the coordinate origin and the observation point. When the dimension of a recovering element along the z axis is small (small \(2\zeta_0\)), we may assume that the phase factor satisfies the condition

\[ \exp \left\{ i \frac{\omega_2}{c} \left[ \frac{1}{\rho_p} + \frac{1}{\rho_3} + \frac{\omega_1}{\omega_2} \left( \pm \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right] \right\} \ll 2\pi, \]
Fig. 3. Schematic diagram for reconstructing focused radiation from a hologram in a waveguide regime.

i.e., we may neglect this component in the phase. In the expression for the reconstructed field, the integrals split, which yields

\[
E_3(x, y, z, t) = \frac{g \omega_2^2 \varepsilon_0 \varepsilon_r}{4 \pi c^2 \rho_0 \rho_r \rho_p \rho_3} \exp\left\{-i \omega_2 t + i \frac{\omega_2}{c} \left[ \rho_p + \frac{\omega_1}{\omega_2} (\pm \rho_0 - \rho_r) \right]\right\} \\
\times \int \varepsilon_x \left[ t - r_3(P, P_3) \right] t(\xi) \exp\left\{i \frac{\omega_2}{2c} (\xi^2 + \eta^2) \left[ \frac{1}{\rho_0} + \frac{\omega_1}{\omega_2} \left( \pm \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right]\right\} \\
- i \frac{\omega_2}{c} \left[ \cos \vartheta_{x0} + \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{x0} - \cos \vartheta_{x0}) \right] - i \frac{\omega_2}{c} \eta \left[ \cos \vartheta_{y0} + \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{y0} - \cos \vartheta_{y0}) \right]\right\} \\
\times \exp\left[i \frac{\omega_2}{c} r_3(P_{z=0}, P_3)\right] d\xi d\eta
\]
The integral with respect to $\zeta$ has the form of the function $(\sin U)/U$, i.e., is close to a $\delta$-function. The position of the reconstructed wave is determined by the maximum condition for a maximum of the function $(\sin U)/U$, i.e., for the integral with respect to $\zeta$:

$$\frac{\omega_1}{c} \zeta_0(\pm \cos \vartheta_{x0} - \cos \vartheta_{xr}) + \frac{\omega_2}{c} \zeta_0(\cos \vartheta_{xp} + \cos \vartheta_{x2}) = 0. \quad (25)$$

Finally, the component $E_3(x, y, z, t)$ of the reconstructed field with the constraint (25), with allowance made for the required resolution, and under the assumption that the reconstruction pulse is approximated by a rectangle is determined by the formula [see (24)]

$$E_3(x, y, z, t) = \frac{gw_2^2 \varepsilon_0 \varepsilon_r}{4\pi c^2 \rho_0 \rho_p \rho_{p_3}} \exp \left\{-i\omega_2 t + i\frac{\omega_2}{c} \left[ \frac{\rho_p}{\rho_0} + \frac{\omega_1}{\omega_2} \left( \pm \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right]\right\} t(u_1 t)$$

$$\times \int \int \exp \left\{ i\frac{\omega_2}{2c} (\xi^2 + \eta^2) \left[ \frac{1}{\rho_p} + \frac{1}{\omega_2} \left( \pm \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right] \right\}$$

$$\times \left\{-i\frac{\omega_2}{c} \xi \left[ \cos \vartheta_{xp} + \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{x0} - \cos \vartheta_{x2}) \right] \right\}$$

$$\times \left\{-i\frac{\omega_2}{c} \eta \left[ \cos \vartheta_{yp} + \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{y0} - \cos \vartheta_{yr}) \right] \right\} \exp \left\{ i\frac{\omega_2}{c} r_3(P_{x=0}, P_3) \right\} \, d\xi \, d\eta. \quad (26)$$

This expression coincides with (18) with accuracy up to insignificant constant factors, i.e., the field structure is not changed in the case where spatial effects are taken into account; however, additional constraint (25) arises.

In view of (25), one can take into account in (24) the spectral composition of the reading pulse [see (16)] in a way similar to that used in (17). Calculations show that the spectrum of a signal is formed in the region where the reconstructed radiation is focused.

The component $E_4(x, y, z, t)$ of the reconstructed field due to the component $\Delta \varepsilon_4(\xi, \eta, \zeta)$ of the permittivity perturbation [see (8), (9)] can be found in a similar way. The field is considered at the point $P_4$ with the coordinates $x, y, z$ in the observation zone at a distance

$$\rho_4 = (x^2 + y^2 + z^2)^{1/2}$$

from the origin of coordinates. In view of (8)–(10), similarly to the case of $E_3(x, y, z, t)$, the field component under consideration is described by the expression

$$E_4(x, y, z, t) = \frac{gw_2^2 \varepsilon_0 \varepsilon_r}{4\pi c^2 \rho_0 \rho_p \rho_{p_4}} \int \varepsilon_p \left( t - \frac{r_4(P, P_4)}{c} \right) t(\xi) \exp \left\{ i\frac{\omega_2}{c} \left[ \rho_p - \frac{\omega_1}{\omega_2} (\pm \rho_0 - \rho_r) \right] \right\}$$

$$+ \frac{i}{2c} (\xi^2 + \eta^2 + \zeta^2) \left[ \frac{1}{\rho_p} - \frac{\omega_1}{\omega_2} (\pm \frac{1}{\rho_0} - \frac{1}{\rho_r}) \right]$$

$$- i\frac{\omega_2}{c} \xi \left[ \cos \vartheta_{xp} - \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{x0} - \cos \vartheta_{x2}) \right]$$

$$- i\frac{\omega_2}{c} \eta \left[ \cos \vartheta_{yp} - \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{y0} - \cos \vartheta_{yr}) \right]$$

$$\times \exp \left\{ -i\omega_2 \left[ t - \frac{r_4(P, P_4)}{c} \right] \right\} \, dV. \quad (26)$$
Let us take into account the assumptions made earlier for the amplitude and phase factors in calculating the field $E_3(x, y, z, t)$. In a similar way,

$$r_4(P, P_4) \approx r_4(P_{s=0}, P_4) = \left[ (\xi^2 - x_4)^2 + (\eta - y_4)^2 + (-z_4)^2 \right]^{1/2}$$

for the amplitude factors and

$$r_4(P, P_4) \approx r_4(P_{z=0}, P_4) + \frac{\zeta^2}{2\rho_4} - \zeta \cos \theta_{4} ,$$

$$\exp \left\{ \frac{i \omega_2}{c} \frac{\zeta^2}{2} \left[ \frac{1}{\rho_p} + \frac{1}{\rho_4} \frac{\omega_1}{\omega_2} \left( \pm \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right] \right\} \ll 2\pi$$

for the factors in the phase expressions, where

$$\rho_4 = \left( x_4^2 + y_4^2 + z_4^2 \right)^{1/2}$$

is the distance between the origin of coordinates and the observation point. Splitting the integrals in a way similar to that used in (24), we obtain

$$E_4(x, y, z, t) = \frac{g \omega_2^2 \varepsilon_0 \varepsilon_r}{4 \pi c^2 \rho_0 \rho_r \rho_p \rho_4} \exp \left\{ -i \omega_2 t - \frac{i \omega_2}{c} \left[ \rho_p - \frac{\omega_1}{\omega_2} (\pm \rho_0 - \rho_r) \right] \right\}$$

$$\times \int \varepsilon_r \left[ t - \frac{r_4(P, P_4)}{c} \right] t(\xi) \exp \left\{ \frac{i \omega_2}{2c} (\xi^2 + \eta^2) \left[ \frac{1}{\rho_p} - \frac{\omega_1}{\omega_2} \left( \pm \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right] \right\}$$

$$- \frac{i \omega_2}{c} \xi \left[ \cos \theta_{4 \xi} - \frac{\omega_1}{\omega_2} (\pm \cos \theta_{z_0} - \cos \theta_{3r}) \right] - \frac{i \omega_2}{c} \eta \left[ \cos \theta_{4 \eta} - \frac{\omega_1}{\omega_2} (\pm \cos \theta_{y_0} - \cos \theta_{3r}) \right]$$

$$\times \exp \left\{ \frac{i \omega_2}{c} r_4(P_{z=0}, P_4) \right\} d\xi d\eta$$

$$\times \int \exp \left\{ -\frac{i \omega_2}{c} \zeta \left[ \cos \theta_{4 \zeta} + \cos \theta_{4} - \frac{\omega_1}{\omega_2} (\pm \cos \theta_{z_0} - \cos \theta_{3r}) \right] \right\} d\zeta , \quad (27)$$

The integral with respect to $\zeta$ yields an additional constraint determining the position of the reconstructed wave and described by the relation

$$-\frac{\omega_1}{c} \zeta_0 (\pm \cos \theta_{z_0} - \cos \theta_{3r}) + \frac{\omega_2}{c} \zeta_0 (\pm \cos \theta_{z_0} + \cos \theta_{3r}) = 0 . \quad (28)$$

The final expression describing the reconstructed field [see (27)] with the constraint (28) and with the shape of the reconstructing pulse approximated by a rectangle whose duration is determined by the resolution can be written in the form

$$E_4(x, y, z, t) = \frac{g \omega_2^2 \varepsilon_0 \varepsilon_r}{4 \pi c^2 \rho_0 \rho_r \rho_p \rho_4} \exp \left\{ -i \omega_2 t - \frac{i \omega_2}{c} \left[ \rho_p - \frac{\omega_1}{\omega_2} (\pm \rho_0 - \rho_r) \right] \right\}$$

$$\times \int_{v_t + \Delta \xi}^{v_t - \Delta \xi} \int_{-\eta_0}^{\eta_0} \exp \left\{ -\frac{i \omega_2}{2c} (\xi^2 + \eta^2) \left[ \frac{1}{\rho_p} - \frac{\omega_1}{\omega_2} \left( \pm \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right] \right\}$$

$$- \frac{i \omega_2}{c} \xi \left[ \cos \theta_{4 \xi} - \frac{\omega_1}{\omega_2} (\pm \cos \theta_{z_0} - \cos \theta_{3r}) \right] - \frac{i \omega_2}{c} \eta \left[ \cos \theta_{4 \eta} - \frac{\omega_1}{\omega_2} (\pm \cos \theta_{y_0} - \cos \theta_{3r}) \right]$$

$$\times \exp \left\{ \frac{i \omega_2}{c} r_4(P_{z=0}, P_4) \right\} d\xi d\eta , \quad (29)$$

70
The component $E_4(x, y, z, t)$ of the reconstructed field corresponds to the field described by Eq. (19) without inclusion of spatial effects in the hologram reconstruction.

Taking into account the spectral composition of the reading pulse [see (16)] in a way similar to that used previously [see (17) and (27)] shows that the system forms the spectrum of a signal in the region of focusing.

Thus, accounting for spatial effects in reconstructing a hologram requires only that additional conditions of image formation be taken into account, namely, the constraint (25) in addition to (13) for the component $E_3(x, y, z, t)$ and the constraint (28) is addition to (15) for the component $E_4(x, y, z, t)$.

### 4. Experimental Scheme for Recording and Reconstructing a Waveguide Hologram and Introducing Amplitude-Phase Information into a Fiber Optics Line

Let us analyze an experimental scheme that includes recording a waveguide hologram with external waves, reconstructing it in a waveguide mode, and focusing the reconstructed wave at the input of a fiber optics link (see Figs. 1 and 3). Concrete realization of the scheme imposes certain restrictions on the structure and propagation directions of the waves involved in recording and reconstructing a hologram. In a conventional analysis, the position of the reconstructed images is determined from the prescribed positions of the object and the sources of reference and reconstructing waves (see, e.g., [18]). We consider another case where the structure and the direction of propagation of the recovering wave $E_p(x, y, z, t)$ [see (10)] and the position of the recovered image $E_3(x, y, z, t)$ [see Eqs. (12), (18), and (26)] or $E_4(x, y, z, t)$ [see Eqs. (14), (19), and (29)] are prescribed. The problem is to determine the structure and direction of propagation of the signal wave $E_0(x, y, z, t)$ of (6) and the reference wave $E_r(x, y, z, t)$ of (7) appropriate for hologram recording. We note that certain restrictions are also imposed on the signal and reference waves.

Let us consider particular types of waves allowing for the fact that the wave sources are located in the $xz$-plane, i.e., the mean directions of propagation of all the waves obey the relationships

$$\begin{align*}
y_i &= 0, \\
\vartheta_{yi} &= \frac{\pi}{2}, \\
\cos \vartheta_{yi} &= 0,
\end{align*}$$

where $i$ specifies the wave: $i = 0, r, p, 3, or 4$.

The signal wave [see (6)] used for hologram recording is a converging (minus sign in the wave equations) or diverging (plus sign in the wave equations) spherical wave $E_0(x, y, z, t)$ with a singularity at the point $P_0$ with the coordinates $x_0, y_0 = 0, z_0$ (see Fig. 2). In the body of a recording element, the wave is described by the expressions

$$\begin{align*}
\cos \vartheta_{x0} &= \frac{x_0}{\rho_0}, \\
\cos \vartheta_{z0} &= \frac{z_0}{\rho_0}, \\
\rho_0 &= (x_0^2 + z_0^2)^{1/2}
\end{align*}$$

and

$$E_0(\xi, \eta, \zeta, t) = \frac{\xi_0}{\rho_0} \exp(-i\omega_1 t) \exp \left\{ \pm i \frac{\omega_1}{c} \left[ \frac{\xi^2 + \zeta^2}{2\rho_0} - \xi \cos \vartheta_{x0} + \zeta \cos \vartheta_{z0} \right] \right\}. \quad (30)$$
The reference wave \( E_r(x, y, z, t) \) [see (7)] used in hologram recording is a diverging spherical wave with the origin at the point \( P_r \) with the coordinates \( x_r, y_r = 0, z_r \) (see Fig. 2). Its direction of propagation is characterized by the direction cosines

\[
\cos \vartheta_{x_r} = \frac{x_r}{\rho_r}, \\
\cos \vartheta_{z_r} = \frac{z_r}{\rho_r}, \\
\rho_r = (x_r^2 + z_r^2)^{1/2}.
\]

In the body of a recording element, the wave is described by the formula

\[
E_r(\xi, \eta, \zeta, t) = \frac{\varepsilon_r}{\rho_r} \exp(-i\omega_1 t) \exp \left\{ \pm \frac{i\omega_1}{c} \left[ \rho_r + \frac{\xi^2 + \zeta^2}{2\rho_r} - \xi \cos \vartheta_{x_r} + \zeta \cos \vartheta_{z_r} \right] \right\}. \tag{31}
\]

The reconstructing wave [see (10)] in the body of a recording element can be considered plane. This is due either to the geometry of wave formation in the single-mode waveguide or to the selection of directions in the wave because of spatial effects in the hologram. The operating conditions of the scheme include propagation of the reconstructing wave in the forward or backward direction along the \( x \) axis, which is described by the constraints

\[
\vartheta_{xp} = 0, \pi \quad \text{and} \quad \cos \vartheta_{xp} = \pm 1; \\
\vartheta_{yp} = \vartheta_{zp} = \frac{\pi}{2} \quad \text{and} \quad \cos \vartheta_{yp} = \cos \vartheta_{zp} = 0.
\]

As a result, the reconstructing wave in the body of a recording element is given by the expression

\[
E_p(\xi, \eta, \zeta, t) = \varepsilon_p \exp(-i\omega_2 t) \exp \left( \mp \frac{i\omega_2}{c} \xi \right). \tag{32}
\]

The minus sign corresponds to the wave propagating in the positive direction of the \( x \) axis, whereas the plus sign corresponds to the wave propagating in the negative direction.

The description of the structure of the waves obtained from a waveguide hologram recorded by waves of type (30) and (31), reconstructed by a wave of type (32), and modulated with an information transparency will be based on the general formulas obtained above [see (12) or (26) and (14) or (29)]. These waves should satisfy the most rigorous conditions. The reconstructed spherical waves should provide focusing of radiation into a point with the coordinates \( x_0, 0, \pm z_0 \) at the input to a communication line. The different signs correspond to opposite positions of the line entrance with respect to the waveguide hologram. On average, the waves propagate approximately in a normal direction with respect to the hologram plane. The angles and direction cosines for the average directions of propagation are given by

\[
\vartheta_{x3} \sim \frac{\pi}{2}, \quad \vartheta_{y3} = \frac{\pi}{2}, \quad \vartheta_{z3} = 0, \pi, \\
\cos \vartheta_{x3} \sim 0, \quad \cos \vartheta_{y3} = 0, \quad \cos \vartheta_{z3} = \pm 1.
\]
for $E_3(x, y, z, t)$, and by

$$\begin{align*}
\vartheta_{x4} &\sim \frac{\pi}{2}, \quad \vartheta_{y4} = \frac{\pi}{2}, \quad \vartheta_{z4} = 0, \pi, \\
\cos \vartheta_{x4} &\sim 0, \quad \cos \vartheta_{y4} = 0, \quad \cos \vartheta_{z4} = \pm 1
\end{align*}$$

for $E_4(x, y, z, t)$.

The resulting reconstructed wave $E_3(x, y, z, t)$ is described, with accuracy up to insignificant constant factors, by the formula [see (18) and (26)]

$$
E_3(x, y, z, t) \approx t(u_{zt} \exp (-i\omega_2 t) \int_{v_{zt} - \Delta \xi}^{v_{zt} + \Delta \xi} \int_{-\eta_0}^{\eta_0} \exp \left\{ i \frac{\omega_2}{2c} \left( \xi^2 + \eta^2 \right) \frac{\omega_1}{\omega_2} \left( \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right\} \\
- i \frac{\omega_2}{c} \xi \left[ \pm 1 + \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{x0} - \cos \vartheta_{xr}) \right] \exp \left\{ i \frac{\omega_2}{c} r_3(P_{z=0}, P_3) \right\} d\xi d\eta. \tag{33}
$$

The conditions for obtaining a converging spherical wave have the form [see (13)]

$$
\frac{1}{\rho_3} = - \frac{\omega_1}{\omega_2} \left( \frac{1}{\rho_0} - \frac{1}{\rho_r} \right), \quad \tag{34}
\cos \vartheta_{x3} = \mp 1 - \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{x0} - \cos \vartheta_{xr}),
$$

with the constraints

$$
\cos \vartheta_{y3} = 0, \\
\cos \vartheta_{x0} = (1 - \cos^2 \vartheta_{x0})^{1/2}, \\
\cos \vartheta_{xr} = (1 - \cos^2 \vartheta_{xr})^{1/2}.
$$

Taking into account spatial effects in hologram reconstruction gives the additional constraint [see (25)]

$$
\omega_1 (\pm \cos \vartheta_{x0} - \cos \vartheta_{xr}) + \omega_2 (\pm 1 + \cos \vartheta_{x3}) = 0, \quad \tag{35}
\cos \vartheta_{x3} = (1 - \cos^2 \vartheta_{x3})^{1/2}.
$$

Similarly, the reconstructed wave $E_4(x, y, z, t)$ with accuracy up to insignificant constant factors is described by the formula [see (19) and (29)]

$$
E_4(x, y, z, t) \approx t(u_{zt} \exp (-i\omega_2 t) \int_{v_{zt} - \Delta \xi}^{v_{zt} + \Delta \xi} \int_{-\eta_0}^{\eta_0} \exp \left\{ i \frac{\omega_2}{2c} \left( \xi^2 + \eta^2 \right) \frac{\omega_1}{\omega_2} \left( \frac{1}{\rho_0} - \frac{1}{\rho_r} \right) \right\} \\
- i \frac{\omega_2}{c} \xi \left[ \pm 1 - \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{x0} - \cos \vartheta_{xr}) \right] \exp \left\{ i \frac{\omega_2}{c} r_4(P_{z=0}, P_4) \right\} d\xi d\eta. \tag{36}
$$

This wave is a converging spherical one if the conditions [see (15)]
The obtained expressions (34) and (37) make it possible to determine the source position \((\rho_r)\) and the direction of propagation (direction cosines) of the reference wave \(E_r(x,y,z,t)\) [see (31)] for a prescribed position of the singular point (the point of a source or focusing) for the signal wave \(E_0(x,y,z,t)\) [see (30)] and, hence, a prescribed average direction of its propagation and for definite frequencies \(\omega_1\) and \(\omega_2\). To avoid confusion in the signs of the direction cosines, we will write out separately the expressions for the reconstructing wave propagating toward positive and negative \(x\) values.

For the reconstructed wave \(E_3(x,y,z,t)\) [see (33)], the direction cosines of the reference wave with respect to the \(x\) axis are determined by the expressions [see (34)]

\[
\frac{1}{\rho_r} = \frac{\omega_1}{\omega_2} \left( \frac{1}{\rho_0} + \frac{1}{\rho_r} \right),
\]

\[
\cos \vartheta_{x4} = \pm 1 + \frac{\omega_1}{\omega_2} (\pm \cos \vartheta_{x0} - \cos \vartheta_{x4})
\]

and the constraints

\[
\cos \vartheta_{y4} = 0,
\]

\[
\cos \vartheta_{z0} = (1 - \cos^2 \vartheta_{x0})^{1/2},
\]

\[
\cos \vartheta_{xr} = (1 - \cos^2 \vartheta_{xr})^{1/2}
\]

hold. Taking into account spatial effects gives the additional constraint [see (28)]

\[
-\omega_1 (\pm \cos \vartheta_{x0} - \cos \vartheta_{xr}) + \omega_2 (\pm 1 + \cos \vartheta_{x4}) = 0,
\]

\[
\cos \vartheta_{x4} = (1 - \cos^2 \vartheta_{x4})^{1/2}.
\]

The obtained expressions (34) and (37) make it possible to determine the source position \((\rho_r)\) and the direction of propagation (direction cosines) of the reference wave \(E_r(x,y,z,t)\) [see (31)] for a prescribed position of the singular point (the point of a source or focusing) for the signal wave \(E_0(x,y,z,t)\) [see (30)] and, hence, a prescribed average direction of its propagation and for definite frequencies \(\omega_1\) and \(\omega_2\). To avoid confusion in the signs of the direction cosines, we will write out separately the expressions for the reconstructing wave propagating toward positive and negative \(x\) values.

For the reconstructed wave \(E_3(x,y,z,t)\) [see (33)], the direction cosines of the reference wave with respect to the \(x\) axis are determined by the expressions [see (34)]

\[
\frac{1}{\rho_r} = \frac{\omega_1}{\omega_2} \frac{1}{\rho_3} + \frac{1}{\rho_0},
\]

\[
\cos \vartheta_{xr} = \frac{\omega_2}{\omega_1} (\cos \vartheta_{x3} + 1) \pm \cos \vartheta_{x0}
\]

if the reconstructing wave propagates in the positive direction of the \(x\) axis and

\[
\frac{1}{\rho_r} = \frac{\omega_1}{\omega_2} \frac{1}{\rho_3} - \frac{1}{\rho_0},
\]

\[
\cos \vartheta_{xr} = \frac{\omega_2}{\omega_1} (\cos \vartheta_{x3} - 1) \pm \cos \vartheta_{x0}
\]

if it propagates in the opposite direction.

For the reconstructed wave \(E_4(x,y,z,t)\) [see (36)], the direction cosines of the reference wave are determined by the relations [see (37)]
\[
\frac{1}{\rho_r} = -\frac{\omega_1}{\omega_2 \rho_4} + \frac{1}{\rho_0},
\]

\[
\cos \vartheta_{xr} = \frac{\omega_2}{\omega_1} (\cos \theta_4 - 1) \pm \cos \vartheta_{x0}
\]

if the reconstructing wave propagates in the positive direction of the \(x\) axis and

\[
\frac{1}{\rho_r} = -\frac{\omega_1}{\omega_2 \rho_4} - \frac{1}{\rho_0},
\]

\[
\cos \vartheta_{xr} = \frac{\omega_2}{\omega_1} (\cos \theta_4 + 1) \pm \cos \vartheta_{x0}
\]

if it propagates in the opposite direction.

In all the cases considered above, the direction cosines with respect to the \(y\) and \(z\) axes are given by the same expressions

\[
\cos \vartheta_{yr} = 0,
\]

\[
\cos \vartheta_{zr} = (1 - \cos^2 \vartheta_{xr})^{1/2}.
\]

The same formulas (34) and (37) can be used to solve one more problem. We can determine the position of the singular point of the signal wave \(E_0(x, y, z, t)\) [see (30)] for a prescribed position of the source and direction of the reference wave \(E_r(x, y, z, t)\) [see (31)].

For the reconstructed wave \(E_3(x, y, z, t)\) [see (33)], the direction cosines with respect to the \(x\) axis and the distance between the origin of coordinates and the singular point for the signal wave are determined by the expressions [see (34)]

\[
\frac{1}{\rho_0} = -\frac{\omega_1}{\omega_2 \rho_3} + \frac{1}{\rho_r},
\]

\[
\pm \cos \vartheta_{x0} = \frac{\omega_2}{\omega_1} (\cos \theta_3 - 1) + \cos \vartheta_{xr}
\]

if the reconstructing wave propagates in the positive direction of the \(x\) axis and

\[
\frac{1}{\rho_0} = \frac{\omega_1}{\omega_2 \rho_3} - \frac{1}{\rho_r},
\]

\[
\pm \cos \vartheta_{x0} = \frac{\omega_2}{\omega_1} (\cos \theta_3 + 1) + \cos \vartheta_{xr}
\]

if it propagates in the opposite direction.

For the reconstructed wave \(E_4(x, y, z, t)\) [see (36)], the direction cosines with respect to the \(x\) axis and the distance between the origin of coordinates and the singular point of the signal wave are determined by the relations [see (37)]
\[ \frac{1}{\rho_0} = \frac{\omega_1}{\omega_2 \rho_4} \frac{1}{\rho_4} + \frac{1}{\rho_4} , \]
\[ \pm \cos \theta_{x_0} = \frac{\omega_2}{\omega_1} (\cos \theta_{x_4} + 1) + \cos \theta_{x_0} , \]

if the reconstructing wave propagates in the positive direction of the z axis and
\[ \frac{1}{\rho_0} = -\frac{\omega_1}{\omega_2 \rho_4} \frac{1}{\rho_4} , \]
\[ \pm \cos \theta_{x_0} = \frac{\omega_2}{\omega_1} (\cos \theta_{x_4} - 1) + \cos \theta_{x_0} , \]

if it propagates in the opposite direction.

In all the cases considered above, the direction cosines with respect to the y and z axes are given by the same expressions
\[ \cos \theta_{y_0} = 0 , \]
\[ \cos \theta_{z_0} = (1 - \cos^2 \theta_{x_0})^{1/2} . \]

Relationships (35) and (38) impose additional restrictions on the geometry of the signal and reference waves. For the direction of the reference wave, this yields in the case of the reconstructed wave \( E_3(x, y, z, t) \) [see (33)] the relations [see (35)]
\[ \cos \theta_{x_3} = +\frac{\omega_2}{\omega_1} (\cos \theta_{x_3} + 1) \pm \cos \theta_{x_0} \] (43)

or
\[ \cos \theta_{x_3} = -\frac{\omega_2}{\omega_1} (\cos \theta_{x_3} - 1) \pm \cos \theta_{x_0} . \] (44)

In the case of the reconstructed wave \( E_4(x, y, z, t) \) [see (36)] we obtain correspondingly [see (38)]
\[ \cos \theta_{x_4} = -\frac{\omega_2}{\omega_1} (\cos \theta_{x_4} + 1) \pm \cos \theta_{x_0} \] (45)

or
\[ \cos \theta_{x_4} = +\frac{\omega_2}{\omega_1} (\cos \theta_{x_4} - 1) \pm \cos \theta_{x_0} . \] (46)

Let us analyze the structure of the reconstructed waves with allowance for the fulfillment of certain conditions [see (39)-(42)] imposed on the geometry of the scheme and determined by the direction of propagation of the reference wave for a prescribed position of the singular point of the signal wave and direction of the reconstructing wave. If the direction of the reference wave satisfies conditions (39) or (40), then the reconstructed wave \( E_3(x, y, z, t) \) [see (33) and (34)] satisfies conditions that indicate formation of a converging spherical wave. In this case, such conditions do not hold for the reconstructed wave \( E_4(x, y, z, t) \) [see (36) and (37)]. Conditions that describe a diverging wave also do not hold for this wave. In all cases, the direction cosine takes a value that is contradictory to the trigonometric definitions. This means that the wave \( E_4(x, y, z, t) \) cannot be reconstructed under conditions (39) or (40). We note that due to the symmetry of the geometrical scheme with respect to the xy plane (the hologram plane), the wave \( E_3(x, y, z, t) \) is reconstructed in both the positive and negative directions of the z axis, which is described by the expression
\[ \cos \theta_{z_3} = \pm 1 . \] (47)
It can similarly be shown that if the direction of the reference wave satisfies conditions (41) or (42), then the reconstructed wave $E_4(x, y, z, t)$ [see (36) and (37)] represents a converging spherical wave. In this case, the reconstructed wave $E_3(x, y, z, t)$ [see (33) and (34)] does not satisfy conditions for a converging spherical wave, and the value of the direction cosine contradicts the trigonometric definitions. Hence, the wave $E_3(x, y, z, t)$ cannot be reconstructed under conditions (41) or (42). Similarly, due to the symmetry of the geometrical scheme with respect to the plane of the waveguide hologram, the reconstructed wave $E_4(x, y, z, t)$ is formed in both the positive and negative directions of the $z$ axis. This is expressed by the formula

$$\cos \theta_4 = \pm 1. \quad (48)$$

Thus, if the reference and reconstructing waves have proper directions, then for a particular signal wave two converging spherical waves are reconstructed that propagate in opposite directions perpendicular to the $xy$ plane. Each of them can be introduced into a fiber communication line to transfer information on a light field that is modulated by a complex function.

This situation can be explained in more detail. The reconstructed component $E_3(x, y, z, t)$ can be focused at the entrance to a fiber optics communication line either in the case of a reference wave with direction of propagation satisfying (39) and a reconstructing wave propagating in the positive direction of the $x$ axis or in the case of a reference wave satisfying (40) and a reconstructing wave propagating in the negative direction of the $x$ axis. The reconstructed component $E_4(x, y, z, t)$ can be focused either in the case of a reference wave satisfying condition (41) and a reconstructing wave propagating in the positive direction of the $x$ axis or in the case of a reference wave satisfying condition (42) and a reconstructing wave propagating in the negative direction of the $x$ axis.

It is of interest to take into account the additional constraints (43)–(46) associated with inclusion of spatial effects in a hologram. These conditions impose additional restrictions on the position of the singular point in a signal wave. For the component $E_3(x, y, z, t)$ reconstructed under fulfillment of conditions (39) and (43), we have

$$\cos \theta_{x_3} = \frac{\omega_2}{\omega_1} (\cos \theta_{z_3} + 1) \pm \cos \theta_{z_0},$$
$$\cos \theta_{x_3} = \frac{\omega_2}{\omega_1} (\cos \theta_{z_3} + 1) \pm \cos \theta_{z_0}.$$

Squaring and adding, we obtain

$$\pm 2[\cos \theta_{x_0}(\cos \theta_{z_3} + 1) + \cos \theta_{z_0}(\cos \theta_{z_3} + 1)] = -\frac{\omega_2}{\omega_1}[3 + 2(\cos \theta_{z_3} + \cos \theta_{z_3})].$$

For the same component $E_3(x, y, z, t)$ reconstructed under fulfillment of conditions (40) and (44), the additional constraint has the form

$$\pm 2[\cos \theta_{x_0}(\cos \theta_{z_3} - 1) - \cos \theta_{z_0}(\cos \theta_{z_3} - 1)] = -\frac{\omega_2}{\omega_1}[3 - 2(\cos \theta_{z_3} + \cos \theta_{z_3})].$$

For the reconstructed component $E_4(x, y, z, t)$, we similarly have

$$\pm 2[\cos \theta_{x_0}(\cos \theta_{z_4} - 1) - \cos \theta_{z_0}(\cos \theta_{z_4} - 1)] = -\frac{\omega_2}{\omega_1}[3 - 2(\cos \theta_{z_4} - \cos \theta_{z_4})]$$

under fulfillment of conditions (41) and (45) and

$$\pm 2[\cos \theta_{x_0}(\cos \theta_{z_4} + 1) + \cos \theta_{z_0}(\cos \theta_{z_4} - 1)] = -\frac{\omega_2}{\omega_1}[3 + 2(\cos \theta_{z_4} - \cos \theta_{z_4})].$$

77
under fulfillment of conditions (42) and (46).

A concrete experimental scheme can be specified according to the given relationships.

5. Experimental Setup and Results

In this section, we consider the principal elements of an experimental setup for the employment of waveguide holograms and the experimental results obtained. It is realized on the basis of the optical scheme considered above (see Sec. 4) and presented in Fig. 1 or 3.

The basic optical elements in the given variant of the method used for forming properly modulated radiation and introducing it into a single-mode optical fiber are waveguide diffraction elements or, in a broader sense, waveguide holograms. As noted above, such holograms combine specific properties of spatial diffraction structures and integrated optical elements with operation in a sequential mode [9, 10].

Using results of [9, 10] as a basis, let us present some characteristics of waveguide holograms that are of interest for the application under consideration. The waveguide holograms were made on the basis of ion-exchange glass waveguides with a detection coating evaporated onto them in the form of amorphous layers of chalcogenide semiconductors or photoresist materials. In particular, we employed an As$_2$S$_3$ semiconductor and an AZ–1350 photoresist with a thickness of the order of 1 μm. Recording was carried out using an He–Cd laser (λ = 0.44 μm). To eliminate attenuation of light due to absorption and scattering in the semiconductor or photoresist layer in reconstructing a hologram, we used in actuality a series of photosensitive segments separated by gaps and intended for recording microholograms. The whole structure represented a macrohologram. To enhance the efficiency and decrease the spatial selectivity, the amplitude-phase pattern recorded in a recording layer could be converted to a relief form by etching the As$_2$S$_3$ layer. In the case of substrates of size 100 mm, this made it possible to record up to 50 microholograms on a waveguide macrohologram [10].

In the experiments, we recorded waveguide focusing gratings of size 25 × 30 mm and a series comprising up to 12 microholograms, each of size 2 mm$^2$, with a separation of 4 mm between their centers, which corresponded to a temporal delay of 20 ps between neighboring microholograms. They were recorded in the scheme (see Fig. 2) involving two spherical waves of type (6), (7) or (30), (31). The parameters $\rho_0, \rho_r, \cos \vartheta_a,$ and $\cos \vartheta_r$ were chosen according to Eqs. (37) or (39)–(42) describing formation of a real image in reconstruction. An analysis of the recording scheme and a calculation of its parameters were given above (see Sec. 4).

One of the basic elements in the schemes under consideration was the source of optical picosecond pulses of coherent radiation used for reading the hologram. We used multisection laser diodes operating in a specific regime of passive Q switching, which provided formation of trains of high-power ultrashort pulses with a repetition frequency of up to several tens of hertz [19]. The advantages of these lasers were high efficiency, low control signals, picosecond pulses at the output, low weight, and small size. One more obvious advantage is associated with the possibility of integrating such laser structures into other optoelectronic and integrated optical elements.

The laser diodes used in our work represented a double mesostripe heterostructure formed on the basis of AlGaAs–GaAs layers [19]. The width of a mesostripe waveguide and the thickness of an active region were equal to 5 μm and 0.1 μm, respectively. Three independent electrical contacts were formed along the optical axis of a laser cavity by a photolithographic method. Under certain conditions of power supply, they served as amplifying and absorbing regions of the laser. An absorbing region 40 μm in length was placed in the middle part of the laser cavity. Emitting regions at the ends of the laser were 200 μm long. The total length of the laser was 500 μm.

The amplifying regions were pumped by positive pulses of electric current with a duration of about 10 ns
and a repetition frequency ranging from 10 kHz to 3 MHz. The amplitude of the pumping pulses was varied in the range of 0.1–1.0 A. A locking voltage of (1–10) V was applied across the middle region of the laser. Under these conditions of power supply, an electric field with an intensity in the range from +10 to −10 V/cm was formed between the absorbing and emitting sections of the laser diode. This electric field favored dispersal of electron–hole pairs in the absorber and provided additional injection of carriers into the emitting regions.

The spectral characteristics of the emission pulses were investigated by means of DFS–12 spectrometer with a resolution of better than 0.08 nm. The dynamics of emission and the temporal parameters were determined with a time resolution of 2 ps using an Agat–SF image converter. The spectrum of emission was found to have an extremely large width $\Delta \lambda \sim 15–20$ nm and an average wavelength $\lambda \sim 840–850$ nm. The dynamics of laser emission depended considerably on the pumping regime.

When operating near the threshold, the lasers emitted a single optical pulse of duration $\tau \sim 5–7$ ps per electric pumping pulse. A typical chronogram of a single emission pulse is shown in Fig. 4. The number of pulses increased with gradual increase in the locking voltage across the absorbing region or with increase in the amplitude of the pumping current. Thus, one can obtain a train of a prescribed number of optical pulses by varying the pumping regime. Each of the pulses in a train had the same energy of approximately 50 pJ. The duration of each pulse was $\tau \approx 3–7$ ps. A comparison of the spectral width $\Delta \lambda_p$ corresponding to the reciprocal of the pulse duration $\tau$ with the experimental spectral width $\Delta \lambda$ yields

$$\Delta \lambda_p = \frac{\lambda^2}{c \tau} \sim 0.34 – 0.80 \text{ nm} \ll \Delta \lambda \sim 15 – 20 \text{ nm},$$

i.e., the lasers under study were characterized by no more than partial mode locking.

More comprehensive data on lasers of this type are given in [20].

From the viewpoint of employing the scheme under consideration for input of information into an optical fiber, the most important feature of the waveguide hologram is the distributed coupling between a waveguide
mode and the diffraction structure. In contrast to a conventional Fresnel hologram, an elementary guided beam interacted sequentially with a linear region of a hologram and, consequently, each portion of the spatial distribution of the output beam carried information about previous portions in the longitudinal direction [see (18), (26), (33) or (19), (29), (36)]. The spatial coherence of the beam did not change noticeably in the process, whereas in the time domain we observed not only chromatic decomposition [see (17)] but also space-time transformation of the temporal front [see (33) or (36)]. When a reading light pulse of duration $\tau \sim 1$ ps, for example, entered the waveguide, it had a dimension of $2\Delta \xi \sim 200$ $\mu$m in the hologram material and it formed a system of beams upon diffraction by a series of waveguide microholograms of length, e.g., 1 mm. These beams were separated both in space, due to information recorded on the hologram, and in time with delays that were proportional to the distance between microholograms.

Upon converting a single reading pulse into a temporal sequence, each of the channels could be separately modulated separately at a frequency not lower than the repetition frequency of the reading pulses. Combining all channels at the entrance to the fiber link provided input of transmitted information.

As mentioned above, the diffraction efficiency of waveguide holograms in the case under consideration depended on the macrohologram dimension along the direction of propagation of the reconstructing waveguide mode. If microholograms were reconstructed sequentially by a single waveguide mode, then, in the case where they had equal diffraction efficiencies $p$, the radiance of the output signals decreased as $(1 - p)^i$, where $i$ is the number of the microhologram. The total light yield $K_s$ was given by

$$K_s = 1 - (1 - p)^n,$$

where $n$ is the number of microholograms constituting the waveguide macrohologram.

In order to extract beams with equal intensities, we had to record microholograms with sequentially increasing diffraction efficiency. For the $(i + 1)$-th microhologram, the diffraction efficiency should be equal to

$$p_{i+1} = \frac{p_i}{1 - p_i}.$$

The dynamic range of the recording medium restricted the diffraction efficiency $p_{\text{lim}}$ of the last hologram. For a fixed length of a macrohologram comprising $n$ microholograms, the radiances of the output beams were determined by the coefficients

$$p_i \prod_{k=1}^{i} (1 - p_k).$$

The total light yield $K_{si}$ was as high as

$$K_{si} = 1 - \prod_{k=1}^{n} (1 - p_k) = 1 - \prod_{k=1}^{n} \left[ 1 - \frac{p_{\text{lim}}}{1 + (n - k)p_{\text{lim}}} \right] = \frac{n p_{\text{lim}}}{1 + (n - 1)p_{\text{lim}}}.$$

To provide precise control of the distribution of diffraction efficiency along a hologram, the macrohologram was recorded by repeatedly exposing individual microholograms in succession.

In most experiments, we used waveguide holograms with the following parameters:

- the spatial period in a series was 4 mm, which corresponded to a time interval of 20 ps between the centers of reconstructed signals;
- the number of channels was equal to six;
- the diffraction efficiencies of the holograms increased sequentially in order to provide output beams of equal intensities.

The actual waveguide holograms had a resolution of 2500–3500 $\text{mm}^{-1}$, and their loss did not exceed $6 \text{ dB/cm}$. 

80
Fig. 5. Structure of light beams reconstructed from a hologram in a waveguide regime: a) a snapshot in the plane of the waveguide hologram; b) a snapshot in a plane positioned close to the Fourier plane; c) a snapshot in the Fourier plane.

Fig. 6. Chronograms of signals reconstructed from two holograms characterized by a time delay of 100 ps by a train of picosecond pulses.
A plane waveguide mode reconstructed six spatial beams propagating in an approximately normal direction in the waveguide plane.

Figure 5 shows snapshots of beams extracted from a hologram: in the waveguide plane (a), in a plane positioned close to the Fourier plane (b), and in the Fourier plane (c).

Figure 6 presents chronograms of signals reconstructed from the two outer microholograms. The pulse train used for the reconstruction is seen on the chronogram. In agreement with the macrohologram geometry, the signal delay equals 100 ps.

Thus, in the Fourier plane we observed the spectrum of a light pulse. The spectral decomposition was caused by dispersion associated with the diffraction structure of the waveguide hologram. In the temporal representation of a signal, a series of pulses with a period of 20 ps was formed. The spectrum of laser emission was observed due to the high chromatic dispersion of the waveguide hologram. In principle, this fact can be used for additional filtration or correction of the temporal shape of pulses because the total spectrum of a signal is connected with partial mode locking of a laser, whereas only part of this spectrum is sufficient to form a picosecond pulse.

6. Conclusion

In summary it may be stated that a waveguide hologram used as a fast scanning device offers promise for introducing information on a modulated light field into a single-mode optical fiber.

We now make some numerical estimates. Below, approximate values will be given. Specific values taken for a particular realization are presented, as a rule, in addition. Let us assume that the reading pulse has a duration of several picoseconds (τ ≈ 5 · 10^{-12} s), which is a reasonable value for mode-locked laser diodes. If this is the case, then the length of a working element 2Δξ along the ξ axis in a waveguide hologram can reach one millimeter:

\[ 2Δξ = \frac{cτ}{n} ≈ 1.0 \text{ mm}, \]

where \( c \) is the speed of light, and \( n ≈ 1.5 \) is the refractive index of the waveguide material. The total length of a macrohologram or, in other terms, the total length of a linear transparency to be read and input into a communication line is of the order of several centimeters, \( L_ξ ≈ 6 \text{ cm} \). This means that the total number of linear elements transmitted through the communication line equals several tens:

\[ N = \frac{L_ξ}{2Δξ} ≈ 60. \]

Of some interest is an estimate of the diffraction by a working element. Let us assume that the emission is focused at a distance about ten centimeters from the waveguide plane, \( ρ_3, ρ_4 ≈ 10 \text{ cm} \). The hologram structure for the waveguide reconstruction regime considered above has a period, \( dξ ≈ 1 \text{ μm} \), of the order of the optical wavelength, \( λ ≈ 0.84-0.85 \text{ μm} \). Under the given conditions, a working element of a hologram comprises several hundred elements of the diffraction structure:

\[ m = \frac{2Δξ}{dξ} ≈ 1000. \]

The diffraction by this structure forms a light spot near the input of the optical fiber. Its dimension \( Δx \) along the \( x \) axis is of the order of 1.0 mm:

\[ Δx = \frac{2ρ_3λ}{2Δξ} ≈ 0.9 \text{ mm}. \]
Focusing in the $y$ direction may be much better. For the specific case of a waveguide hologram of length of the order of one centimeter along the $y$ axis, $2\Delta \eta \simeq 1$ cm, the spot size $\Delta y$ in the input plane of an optical fiber is hundredths of a millimeter:

$$\Delta y = \frac{2\rho_3 \lambda}{2\Delta \eta} \simeq 0.048 \text{ mm.}$$

The geometrical dimension of the spectrum in the recording zone can be estimated in a similar way. We will assume, as above, that there is a diffraction holographic structure with a period, $d \xi \simeq 1$ $\mu$m, of the order of the optical wavelength and that the focus is located at a distance of about ten centimeters, $\rho_3, \rho_4 \simeq 10$ cm. For the experimentally determined magnitude of the spectrum, $\Delta \lambda \sim 15-20$ nm, its geometrical dimension equals a few millimeters:

$$\Delta x = \frac{\Delta \lambda \rho_3}{d \xi} \sim (1.5-2.0) \text{ mm.}$$

For the width of the spectrum corresponding to the reciprocal of the pulse duration, $\Delta \lambda_p \sim 0.34-0.80$ nm, its geometrical dimension is equal to

$$\Delta x = \frac{\Delta \lambda_p \rho_3}{d \xi} \sim (34-80) \mu$m.

Thus, in the case of rather accurate focusing along the $y$ axis and an input diameter of a single-mode fiber of the order of ten microns, $d_f \simeq 10$ $\mu$m, losses of light are rather high and a signal is attenuated by a factor of several hundred:

$$\frac{I_s}{I_d} = \frac{S_{xy}}{\pi d_f^2/4} \simeq 500.$$

It is reasonable to employ a photon converter for matching the size of a light spot $S_{xy}$ with the input area of the fiber entry $\pi d_f^2/4$ in order to reduce losses of light.

Let us also estimate the rate of information input into a fiber optics link. For a unit information element of size $2\Delta \xi$ along the $\xi$ axis, $2\Delta \xi \approx 1.0$ mm, the readout time $\Delta t$ coincides with the duration of a reading pulse, which yields a reading rate $V$ of the order of $10^{11}$ elements per second:

$$V = \frac{c}{2\Delta \xi n} \simeq 2 \cdot 10^{11} \text{ elements/s.}$$

The total readout time $\Delta T$ for a linear transparency with $N$ elements is of the order of tenths of a nanosecond:

$$\Delta T = \frac{N}{V} = \frac{L_x}{V 2\Delta \xi} \simeq 0.3 \cdot 10^{-9} \text{ s.}$$

The results and estimates presented above show the feasibility of practical realization of a system for input of phase- and amplitude-modulated information into a fiber optics link with subsequent transmission of this information.

References