ISSN 1063-7842, Technical Physics, 2006, Vol. 51, No. 8, pp. 1060–1068. © Pleiades Publishing, Inc., 2006.
Original Russian Text © E.I. Golant, K.M. Golant, 2006, published in Zhurnal Tekhnicheskoĭ Fiziki, 2006, Vol. 76, No. 8, pp. 99–106.

New Method for Calculating the Spectra and Radiation Losses of Leaky Waves in Multilayer Optical Waveguides

E. I. Golant and K. M. Golant
OOO Biznes–Yunitek, Moscow, 119992 Russia
e-mail: eugenegolant@b-tech.ru
Received January 23, 2006

Abstract—The efficiency of a new method for calculating the spectrum and attenuation coefficient of leaky electromagnetic modes is demonstrated with multilayer planar optical waveguides the guiding properties of which are determined by antiresonant reflection from the multilayer cladding (antiresonant reflecting optical waveguides) rather than by total internal reflection from the core–cladding interface as in standard optical waveguides. The new method applies to calculation of both electromagnetic modes in dielectric waveguides and electron quantum states in multibarrier semiconductor heterostructures. The characteristics of multilayer waveguides calculated by the new method are compared with published data obtained from a complex dispersion relation by the transfer matrix method. As an example, the wavelength dependence of the radiation losses for the first TE mode of a planar optical waveguide containing 52 pairs of layers is calculated.

PACS numbers: 12.20.Ds, 42.25.Bs
DOI: 10.1134/S1063784206080160

INTRODUCTION

Planar optical waveguides where an optical wave propagating through the core antiresonantly reflects from the multilayer cladding (antiresonant reflecting optical waveguides, ARROWs) have been widely studied and used since late in the 1980s. Use of ARROWs at that time was aimed at considerably reducing the electromagnetic energy leakage into the substrate with a refractive index much higher than that of the core [1]. Since then, these waveguides have found wide application in components of optical integrated circuits, such as lasers, sensors, couplers, and polarizers. Simultaneously, a variety of numerical, analytical, and semianalytic methods have been developed to simulate ARROW structures [2–9]. All the numerical methods currently available are based on finding complex roots of the dispersion relation with the transfer matrix, which relates the wave function to its derivative in boundary sections of the structure. This approach has two significant drawbacks. First, computation of the transfer matrix complex coefficients by directly multiplying the transfer matrices of individual layers causes roundoff error accumulation and even a small distortion of the dispersion relation coefficients introduces a serious error into the complex propagation constant. This effect can qualitatively be explained as follows: the presence of exponentially growing components in a general solution to the initial differential problem breaks the numerical stability of the matrix product, as was rigorously proved in [10]. Second, numerical calculation of the complex roots of the transcendental dispersion relation is very involved [7, 8]. Both difficulties aggravate dramatically with increasing number N of layers in the waveguide cladding; therefore, simulations, as a rule, have been restricted to structures with no more than five pairs of layers. At the same time, simulation of optical waveguides that contain tens and hundreds of layers is becoming increasingly important in view of advances in the ARROW technology. For example, an efficient plasma-chemical deposition method was used to fabricate oxynitride glass structures with a cladding consisting of more than 100 pairs of layers [11]. The equations used to describe optical ARROWs are also applicable to quantum states of electrons in semiconductor heterostructures, e.g., coherent-electron-transport cascade quantum lasers [12], which may also contain several hundreds of cascade quantum potential wells and barriers. In this paper, we propose a new approach to characterize leaky waves in multilayer optical waveguides and quantum states of electrons in multilayer heterostructures with coherent electron transport. This approach is synopsized in [13].

NEW APPROACH TO CALCULATION OF ATTENUATION COEFFICIENTS

This approach is convenient to set forth in terms of the wave equation describing the variation of optical wave components along the x coordinate, i.e., in the direction orthogonal to the direction z of wave propagation,

$$\frac{d^2\psi}{dx^2} + [k(x)]^2\psi = 0. \quad (1)$$

Here, x is the transverse coordinate, $k(x) = \sqrt{[n(x)]^2 k_0^2 - \gamma^2} = k_0\sqrt{[n(x)]^2 - n_{\text{eff}}^2}$ is the transverse wavevector of a planar waveguide with refractive index $n(x)$, $\gamma = \beta - i\alpha$ is the longitudinal (generally complex) propagation constant of the wave traveling in the z direction with phase factor $\exp[i(\omega t - \gamma z)]$, $k_0 = \omega/c$ is the wavevector of free space, n_{eff} is the effective refractive index, ω is the circular frequency c is the speed of light in free space, and $\psi = E_y$ for TE waves and H_y for TM waves.

Propagation constant γ and its related effective refractive index n_{eff} are eigenvalues of wave equation (1). The true modes of the waveguide are real-valued solutions to Eq. (1), which have the form of standing waves. The behavior of these solutions (modes) depends on a relationship between the effective index and index $n(\infty)$ of the medium around the waveguide (air, substrate, etc.).

When $n_{\text{eff}} > n(\infty)$, wavevector $k(\infty)$ is a pure imaginary and the physically meaningful component of the solution exponentially decays at infinity; i.e., we are dealing with ordinary propagating modes. A dispersion relation yields a discrete spectrum of effective indices of these modes.

When $n_{\text{eff}} < n(\infty)$, wavevector $k(\infty)$ is real and real solutions at infinity have the form of standing waves, which can be represented as a superposition of two counterpropagating waves,

$$\begin{aligned}\psi(x) &= A\cos[k(\infty)x] + B\sin[k(\infty)x] \\ &= Re^{ik(\infty)x} + Se^{-ik(\infty)x},\end{aligned} \quad (2)$$

where $R = \frac{A - iB}{2}$ and $S = \frac{A + iB}{2}$. These solutions cannot be normalized, because they do not vanish at infinity. They are referred to as radiating modes, since they carry away (dissipate) the energy of the wave in the transverse direction. Simultaneously, the same amount of energy comes from the outside. The corresponding solutions are regarded as having no physical meaning, because they include a wave incoming from infinity. However, these solutions can be used to accurately calculate the attenuation of weakly radiating modes.

The spectrum of radiating modes is continuous, i.e., contains an infinite number of such modes. Each value of effective index $n_{\text{eff}} < n(\infty)$ can be assigned to a particular radiating mode. When the waveguide core is sufficiently wide, these modes (waves) are almost plane. The wider the core, the smaller the portion of the wave energy that is dissipated in the transverse direction (e.g., through the multilayer cladding) and the lower the attenuation coefficient.

At fixed waveguide dimensions and frequency ω of the optical wave, the attenuation constant α versus effective index dependence may exhibit sharp minima, which correspond to the longest-lived modes. These modes are called leaky modes. The goal of our calculation is to determine the spectrum of these modes.

The traditional approach to characterization of leaky modes at $n_{\text{eff}} < n(\infty)$ ignores waves incoming from infinity in (2), leaving only outgoing waves like

$$\psi(x) = Se^{-ik(\infty)x} \text{ or } \psi(x) = Re^{ik(\infty)x}. \quad (3)$$

The solutions then becomes complex-valued, and the roots of the dispersion relation determine complex propagation constant $\gamma = \beta - i\alpha$ and effective index n_{eff} of a leaky mode.

The solutions obtained have a discrete spectrum of modes, but these are not true modes, because they cannot exist without power delivery (excitation) even if the material is perfectly lossless. However, these untrue modes will aid us much in analysis of radiation losses, with an accuracy the higher, the lower the losses. Thus, the definition of radiation losses that relies upon real part α of complex propagation constant γ of the leaky mode is as approximate as nontrivial loss analysis proposed in this paper, which identifies leaky waves with the longest-lived radiating modes. This point should be emphasized.

In the standard approach to evaluating the parameters of a multilayer waveguide, which is based on multiplying the transfer matrices of individual layers [14], a relationship is sought between the boundary wave functions (intensities). The relationship found, together with the condition that only outgoing waves exist, is used to derive a transcendental dispersion relation in complex propagation constant $\gamma = \beta - i\alpha$ of the leaky wave.

Recall the disadvantages of this method once again. The multiplication of transfer matrices raises an error in the coefficients of the dispersion relation, the roots of which are very sensitive to this error. Qualitatively, this is because the presence of exponentially growing components in a general solution to the initial differential problem breaks the stability of computation. Also, the search for complex roots of a complicated transcendental equation is a challenge. Roots can be missed, solutions may be false, etc. These difficulties aggravate dramatically with increasing number of waveguide layers. We are unaware of publications in which leaky modes are analyzed in waveguides with more than ten layers.

To avoid the above difficulties, we developed a new method for calculating the leaky wave spectrum of multilayer waveguides [13]. In this method, real propagation constant β (effective index $n_{\text{eff}} = \beta c/\omega$) and attenuation coefficient α of radiating modes (2) are calculated separately. Continuously varying the effective index (the radiating mode spectrum is continuous), we find the dependence $\alpha(n_{\text{eff}})$ with minima corresponding to leaky modes. Thus, the search for the roots of a complicated transcendental equation on the complex plane is replaced by a simpler problem of searching for local minima of a real function of one variable. To implement this idea, a new technique for calculating the attenuation coefficient is necessary. The basic assumption made in the standard approach to calculating the attenuation coefficient by solving the dispersion relation on the complex plane is that the wave function (intensity) in a waveguide that is homogeneous in the z direction exponentially decays as $z \longrightarrow \infty$ and, consequently, exponentially grows as $z \longrightarrow -\infty$. It is clear that this model is valid only when the attenuation coefficient is smaller than the reciprocal wavelength.

Another model can also be proposed in this approximation. Suppose that perfectly reflecting planes are placed in the sections $x = \pm a$ of the waveguide at sufficiently large distance a from its center that prevent electromagnetic energy leaking outside the system. The problem then becomes well-posed and self-adjoint, with the real eigenvalues and the eigenfunctions having the form of standing waves (2), which are representable as a superposition of two counterpropagating waves. The energy transferred by either partial wave per unit time is taken as the power radiated in the transverse direction, i.e., the loss power. To substantiate this assumption, we instantaneously remove the reflecting planes to see that exactly this partial power will be transferred through the open boundaries at the initial instant. The results obtained with such an approach coincide with published results obtained earlier by traditional methods, which adds to the validity of the above approximation. It should be noted once more that both methods of loss evaluation are approximate and are applicable if the attenuation on the wavelength scale is low.

EXACT DIFFERENCE SCHEME

The problem of finding modes of a multilayer planar optical waveguide reduces to solving the stationary wave equation (Schrödinger equation) for a medium with a piecewise constant (graded) index (potential energy) profile,

$$\frac{d^2\psi}{dx^2} + k_j^2\psi = 0, \tag{4}$$

where $k_j = \sqrt{n_j^2 k_0^2 - \gamma^2}$ is the transverse wavevector of a jth section of the planar waveguide with constant refractive index n_j.

At the boundaries between layers with different refractive indices, the following coupling conditions must be met:

$$\psi(x_{j+}) = \psi(x_{j-}), \quad \frac{1}{\mu_j}\frac{d\psi}{dx}\bigg|_{x_{j+}} = \frac{1}{\mu_{j-1}}\frac{d\psi}{dx}\bigg|_{x_{j-}}, \tag{5}$$

where

$$\mu_j = \begin{cases} 1, & \text{for TE waves} \\ n_j^{-2}, & \text{for TM waves.} \end{cases}$$

The plus sign refers to the layer that lies in the positive direction from point j; the minus sign, to the one lying in the negative direction.

In the case of electron waves in semiconductor heterostructures, $\mu_j = \frac{m_j^*}{m_0}$, where m_j^* is the effective mass of an electron and m_0 is the mass of a free electron. This formula assumes that the waveguiding structure is bounded by half-spaces with refractive indices n_s and n_c, the positive x direction being from n_s to n_c.

For $x_j \le x \le x_{j+1}$, a general solution to Eq. (4) can be written as

$$\begin{aligned}\psi(x) &= A_j\cos[k_j(x-x_j)] + B_j\sin[k_j(x-x_j)] \\ &= R_j e^{ik_j(x-x_j)} + S_j e^{-ik_j(x-x_j)},\end{aligned} \tag{6}$$

where $R_j = \frac{A_j - iB_j}{2}$ and $S_j = \frac{A_j + iB_j}{2}$. Using coupling conditions (5), one can obtain explicit expressions for the coefficients of matrices T_j,

$$T_j = \begin{bmatrix} \cos(k_j d_j) & \frac{\mu_j}{k_j}\sin(k_j d_j) \\ -\frac{k_j}{\mu_j}\sin(k_j d_j) & \cos(k_j d_j) \end{bmatrix}. \tag{7}$$

These coefficients relate wave function ψ to continuous quantity $\mu_j^{-1}\psi' = \mu_j^{-1}\frac{d\psi}{dx}$ in adjacent sections,

$$\begin{bmatrix} \psi_{j+1} \\ \mu_{j+1}^{-1}\psi'_{j+1} \end{bmatrix} = T_j \begin{bmatrix} \psi_j \\ \mu_j^{-1}\psi'_j \end{bmatrix}, \tag{8}$$

where $d_j = x_{j+1} - x_j$ is the thickness of layer j.

Multiplication of matrices T_j yields transfer matrix T,

$$T = T_{N-1}\dots T_2 T_1 = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}.$$

Matrix T relates the wave function to its derivative in the boundary sections,

$$\begin{bmatrix}\psi_c\\ \mu_c^{-1}\psi'_c\end{bmatrix} = T\begin{bmatrix}\psi_s\\ \mu_s^{-1}\psi'_s\end{bmatrix}, \qquad (9)$$

where $\psi_s = \psi_1$, $\psi_c = \psi_N$, $\mu_s^{-1}\psi'_s = \mu_1^{-1}\psi'_1$, and $\mu_c^{-1}\psi'_c = \mu_N^{-1}\psi'_N$.

Setting $\psi(x) = R_s e^{ik_s(x-x_1)}$ at $x < x_1$ and $\psi(x) = S_c e^{-ik_c(x-x_N)}$ at $x > x_N$ (i.e., assuming that only outgoing waves, which decay when ik is real and positive, exist in the bounding half-spaces), we can easily derive from (6) a dispersion relation for complex longitudinal propagation constant γ,

$$\mu_c^{-1}ik_c t_{11} - \mu_s^{-1}\mu_c^{-1}k_s k_c t_{12} + t_{21} + \mu_c^{-1}ik_s t_{22} = 0. \qquad (10)$$

The roots of this equation, $\gamma = \beta - i\alpha$, yield desired real propagation constant β and attenuation constant α of waveguide leaky modes. The above approach is used, with slight variations, in most works devoted to numerical simulation of modes in ARROW structures.

It should be noted that Eqs. (7) and (8) can be written in terms of a difference scheme relating values ψ_{j-1}, ψ_j, and ψ_{j+1} of the wave function at three points,

$$\begin{aligned}&\left[\mu_j^{-1}\frac{k_j d_j}{\sin(k_j d_j)}\frac{\psi_{j+1}-\psi_j}{d_j} - \mu_{j-1}^{-1}\frac{k_{j-1}d_{j-1}}{\sin(k_{j-1}d_{j-1})}\frac{\psi_j-\psi_{j-1}}{d_{j-1}}\right]\frac{2}{d_j+d_{j-1}}\\ &+\left[\mu_j^{-1}k_j^2\frac{\tan(k_j d_j/2)}{k_j d_j/2} + \mu_{j-1}^{-1}k_{j-1}^2\frac{\tan(k_{j-1}d_{j-1}/2)}{k_{j-1}d_{j-1}/2}\right]\psi_j = 0.\end{aligned} \qquad (11)$$

This difference scheme is routinely applied to approximate Eq. (1) subject to coupling conditions (2). In addition, since $\frac{k_j d_j}{\sin(k_j d_j)} \longrightarrow 1$ and $\frac{\tan(k_{j-1}d_{j-1}/2)}{k_{j-1}d_{j-1}/2} \longrightarrow 1$ as $kd \longrightarrow 0$, scheme (8) in this limit directly passes into the standard second-order difference scheme in $\max(d_j)$ for Eqs. (4) and (5). However, unlike the standard scheme, the scheme being considered is exact for a graded refractive index in the transverse direction; i.e., it is algebraically equivalent to Eqs. (7) and (8). It is in this sense that our finite-difference scheme can be considered exact. An important advantage of writing Eqs. (7) and (8) in form (11) is that Eqs. (11) subject to a variety of different boundary conditions can be solved by extremely efficient and numerically stable methods intended for the solution of difference equations with tridiagonal matrices, in particular, the adjoint counter sweep method.

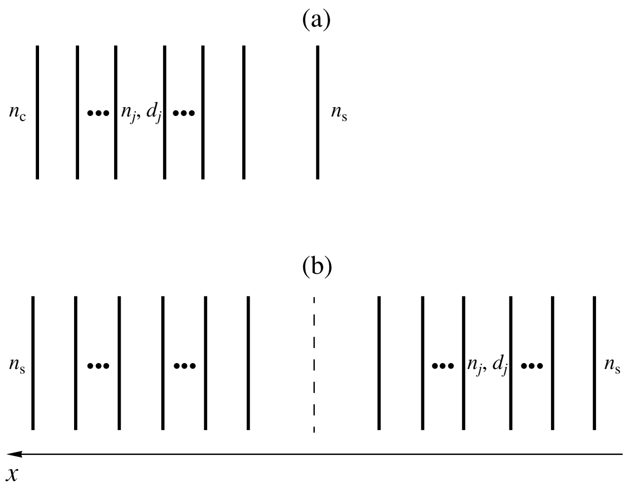

Fig. 1. Schematic representation of layers in (a) asymmetric and (b) symmetric ARROWs.

In this work, we use scheme (11) to numerically simulate two types of waveguide structures: asymmetric structures with energy leakage into the substrate, $n_c > \beta/k_0 > n_s$ (Fig. 1a), and symmetric structures with energy leakage into both the cladding and substrate, $n_c = n_s > \beta/k_0$ (Fig. 1b). In both cases, arbitrary nonzero values of wave function ψ_0 and its derivative ψ'_0 are specified in the section $j = 0$ at the waveguide–substrate (half-space n_s) interface. For the symmetric structures, it suffices to perform calculations for half the structure from the interface to the center of the core and specify a zero wave function (for antisymmetric modes) or a zero derivative of the wave function (for symmetric modes) at the center of the core,

$$\psi_N = 0 \text{ or } \psi'_N = \frac{\psi_N\cos(k_{N-1}d_{N-1}) - \psi_{N-1}}{d_{N-1}} \times \frac{k_{N-1}d_{N-1}}{\sin(k_{N-1}d_{N-1})}\frac{\mu_{N-1}}{\mu_N} = 0. \qquad (12)$$

In the case of the asymmetric structures with total internal reflection from the cladding, the wave function decays in the cladding exponentially with exponent $\kappa = \sqrt{\gamma^2 - n_c^2 k_0^2}$, so that the boundary condition on the waveguide–substrate interface takes the form $\psi_N - \kappa_c\psi'_N = 0$. Using this condition to express the derivative through its values at two adjacent points, we readily obtain a difference boundary condition from Eqs. (7)

and (8),

$$\left[\cos(k_{N-1}d_{N-1}) - \frac{\mu_{N-1}}{\mu_N}\kappa_c d_{N-1}\frac{\sin(k_{N-1}d_{N-1})}{k_{N-1}d_{N-1}}\right]\psi_N - \psi_{N-1} = 0. \quad (13)$$

Complementing system of equations (11) by boundary conditions (12) or (13), we come to a closed system of N linear algebraic equations in N unknowns with a real symmetric tridiagonal matrix. The resulting system can be advantageously solved by one of the methods developed for such systems.

We solve this system of equations under the assumptions that attenuation is absent ($\alpha = 0$) and that wave function ψ_N is real. Thus, a solution to wave equation (1) is sought in the form of standing waves, i.e., in form (6) with real coefficients A_j and B_j and, accordingly, complex conjugate coefficients R_j and S_j. As a consequence of these assumptions, transverse wavevectors $k_j = \sqrt{n_j^2k_0^2 - \beta^2}$ take either real or imaginary values and so all the coefficients of the systems (11), (12) or (11), (13) are real. As a result, given transverse propagation constant β and real values of wave function ψ_0 (or its derivative ψ'_0) at the boundary, the solution of the system (11), (12) or (11), (13) yields values of the wave function at all other points (sections) of the waveguiding structure that are also real.

CALCULATION OF ATTENUATION

This approach implies that electromagnetic power is supplied through one or both boundaries with the outer half-spaces to provide lossless wave propagation. It is clear that the supplied power is equal to the power radiated by the wave into the environment. Let the latter power per unit radiating area be denoted as P_x and the power transferred by the wave in the z direction inside the waveguide (per unit length in the direction perpendicular to the xz plane) P_z. Ratio $P_x(\beta)/P_z(\beta) = \alpha$ has the meaning of the attenuation coefficient of the given mode and varies with β. Minima in the dependence $P_x(\beta)/P_z(\beta)$ correspond to the longest-lived modes, for which the conditions for antiresonant reflection from the cladding of the ARROW are most favorable.

Thus, the complicated problem of searching for complex roots of the transcendental dispersion relation reduces to a much simpler problem of searching for minima of a real function of one variable. Power P_x radiated by the wave into the substrate can be found by applying the Poynting theorem to the wave function (3) component that corresponds to the leaky wave, $R_s e^{ik_s(x-x_1)}$,

$$P_x = \begin{cases} (\varepsilon_0/2)c(k_s/k_0)R_sR_s^* \\ R_sR_s^* = (\psi_0^2 + \psi_0'^2/k_s^2)/4TE \\ Z_0n_s^2(\varepsilon_0/2)c(k_s/k_0)R_sR_s^* \\ R_sR_s^* = [\psi_0^2 + (n_0^2/n_s^2)\psi_0'^2/k_s^2]/4TM, \end{cases} \quad (14)$$

where Z_0 is the impedance of free space. The power density radiated into the cladding at $\beta/k_0 \le n_c$ can be found in a similar fashion. The power transferred by the wave in the z direction in layer j of the waveguide is

$$P_{zj} = \begin{cases} (\varepsilon_0/2)c\dfrac{\beta}{k_0}\displaystyle\int_{x_j}^{x_{j+1}}\psi^2(x)dx\,TE \\ \dfrac{Z_0}{n_j^2}(\varepsilon_0/2)c\dfrac{\beta}{k_0}\displaystyle\int_{x_j}^{x_{j+1}}\psi^2(x)dx\,TM, \end{cases} \quad (15)$$

where

$$\int_{x_j}^{x_{j+1}}\psi^2(x)dx = \frac{d_j}{2}\left[A_j^2\left(1 + \cos k_jd_j\frac{\sin k_jd_j}{k_jd_j}\right) + B_j^2\left(1 - \cos k_jd_j\frac{\sin k_jd_j}{k_jd_j}\right) + A_jB_j\sin k_jd_j\frac{\sin k_jd_j}{k_jd_j}\right],$$

$$A_j = \psi_j, \quad B_j = \psi'_j/k_j.$$

The total power transferred by the wave is a sum of all P_{zj}, $P_z = \sum_{j=0}^{N} P_{zj}$.

It should be noted that Liu et al. [2] proposed a method that makes it possible to calculate mode parameters in planar ARROWs without searching for the roots of the dispersion relation on the complex plane. This method assumes that the field in the half-spaces outside the waveguide is described by standing waves. Based on this assumption and the (complex) transfer matrix method, approximate dispersion relations were derived, all roots of which are real. Losses α associated with energy leakage were found in [2] by numerical differentiation of a transfer matrix element with respect to β.

Our approach differs from that suggested in [2] in two basically important points. First, instead of constructing a dispersion relation with the help of the complex transfer matrix, we numerically solve the wave equation by a stable real-valued exact finite-difference method. Second, our method of calculating the leakage power calculates attenuation coefficient α without resorting to such procedures as numerical differentia-

Table 1

ARROW-A: $n_s = 3.5$, $n_1 = 1.45$, $n_2 = 3.5$, $n_3 = 1.45$, $n_c = 1.0$; $d_1 = 4.0$, $d_2 = 0.1019$, $d_3 = 2.0985$, $\lambda_0 = 1.3$ μm [3–5]						
Mode	Present method		Reference [2]		Reference [2] (exact)	
	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm
TE_1	1.4417085	0.25	1.4417085	0.25	1.4417085	0.25
TE_2	1.417524200	340.3	1.41798	270	1.4176	407

Table 2

ARROW-B: $n_s = 3.85$, $n_1 = 1.54$, $n_2 = 1.46$, $n_3 = 1.54$, $n_c = 1.0$; $d_1 = 4$, $d_2 = 0.3$, $d_3 = 2$, $\lambda_0 = 0.633$ μm [6]						
Mode	Present method		Reference [2]		Reference [2] (exact)	
	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm
TE_1	1.538252749	0.1083	1.5382528	0.11	0.5382527	0.11
TE_2	1.53368551	97.48	1.5336896	95	1.5336856	98

Table 3

Two sided ARROW 1: $n_s = 3.8$, $n_1 = 1.46$, $n_2 = 2.3$, $n_3 = 1.46$, $n_4 = 2.3$, $n_5 = 1.46$, $n_c = 3.8$; $d_1 = 2$, $d_2 = 0.088$, $d_3 = 0.088$, $d_4 = 4$, $\lambda_0 = 0.633$ μm						
Mode	Present method		Reference [2]		Reference [2] (exact)	
	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm
TE_1	1.457852273	0.1052	1.4578523	0.11	1.4578523	0.11
TE_2	1.451845190	96.87	1.4518589	76	1.4518454	98

Table 4

Two sided ARROW 2: $n_s = 3.16$, $n_1 = 3.55$, $n_2 = 3.16$, $n_3 = 3.55$, $n_4 = 3.16$, $n_5 = 3.55$, $n_6 = 3.16$, $n_7 = 3.55$, $n_c = 3.16$; $d_1 = 0.237$, $d_2 = 2$, $d_3 = 0.237$, $d_4 = 4$, $d_5 = 0.237$, $d_6 = 2$, $d_7 = 0.237$, $\lambda_0 = 1.55$ μm						
Mode	Present method		Reference [2]		Reference [2] (exact)	
	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm
TE_1	3.154049689	0.5266	3.1540497	0.53	3.1540497	0.53
TE_2	3.139384329	110.2	3.1393856	103	3.1393856	113

Table 5

Two sided ARROW 3: $n_s = 1.46$, $n_1 = 2.3$, $n_2 = 1.46$, $n_3 = 2.3$, $n_4 = 1.46$, $n_5 = 2.3$, $n_6 = 1.46$, $n_c = 3.5$; $d_1 = 0.089$, $d_2 = 2$, $d_3 = 0.089$, $d_4 = 4$, $d_5 = 0.089$, $d_6 = 2$, $\lambda_0 = 0.6328$ μm						
Mode	Present method		Reference [2]		Reference [2] (exact)	
	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm	β/k_0	loss, dB/cm
TE_1	1.457855835	0.06238	1.4578558	0.06	1.4578558	0.06
TE_2	1.451870544	62.35	1.4518726	49	1.4518551	58

tion or numerical evaluation of the resonance curve width, which noticeably degrade the α calculation accuracy.

COMPARISON OF RESULTS

In order to estimate the accuracy of our method, we used it to calculate the propagation constant (effective

Table 6

3-layer ARROW waveguide: $n_s = 3.85$, $n_1 = 1.46$, $n_2 = 2.30$, $n_3 = 1.46$, $n_c = 1.0$; $d_1 = 6.3\lambda_0$, $d_2 = 0.142\lambda_0$, $d_3 \approx 3.15\lambda_0$, $\lambda_0 = 0.6328$ μm				
Mode	Present method		Reference [7]	
	β/k_0	α/k_0	β/k_0	α/k_0
TE_1	1.457941265	$5.419587209 \times 10^{-8}$	1.45794	5.4189×10^{-8}
TE_2	1.451919115	$5.199112471 \times 10^{-5}$	1.45192	5.2871×10^{-5}
TE_3	1.451174944	$1.885212344 \times 10^{-4}$	1.45117	1.9203×10^{-4}
TE_4	1.441371363	$4.379243431 \times 10^{-6}$	1.44137	4.3745×10^{-6}
TE_5	1.427413890	$2.102732713 \times 10^{-4}$	1.42741	2.1317×10^{-4}
TE_6	1.424450980	$7.516176088 \times 10^{-4}$	1.42445	7.6673×10^{-4}
TE_7	1.407680313	$3.368141566 \times 10^{-5}$	1.40768	3.3582×10^{-5}
TE_8	1.385654140	$4.822274684 \times 10^{-4}$	1.38565	4.8967×10^{-4}
TE_9	1.379006390	$1.688517962 \times 10^{-3}$	1.37900	1.7263×10^{-3}
TE_{10}	1.355673200	$1.293284387 \times 10^{-4}$	1.35567	1.2860×10^{-4}
TE_{11}	1.325099450	$8.817196572 \times 10^{-4}$	1.32510	8.9350×10^{-4}
TE_{12}	1.313214440	$3.018813082 \times 10^{-3}$	1.31320	3.0948×10^{-3}
TE_{13}	1.283292450	$3.564870128 \times 10^{-4}$	1.28330	3.5311×10^{-4}
TE_{14}	1.243206300	$1.431140516 \times 10^{-3}$	1.24321	1.4438×10^{-3}
TE_{15}	1.224199440	$4.815467475 \times 10^{-3}$	1.22418	4.9486×10^{-3}
TE_{16}	1.187203960	$8.195186046 \times 10^{-4}$	1.18720	8.0634×10^{-4}
TE_{17}	1.135928490	$2.157567629 \times 10^{-3}$	1.13592	2.1513×10^{-3}
TE_{18}	1.107020710	$7.276317705 \times 10^{-3}$	1.10700	7.4694×10^{-3}
TE_{19}	1.062409230	$1.733843321 \times 10^{-3}$	1.06241	1.6648×10^{-3}

index) and attenuation of TE modes in planar optical waveguides the parameters of which was evaluated with other different methods [2, 7, 8].

Tables 1–5 compare the parameters of the first two TE modes of the available ARROWs that were calculated by our method with those borrowed from [2].

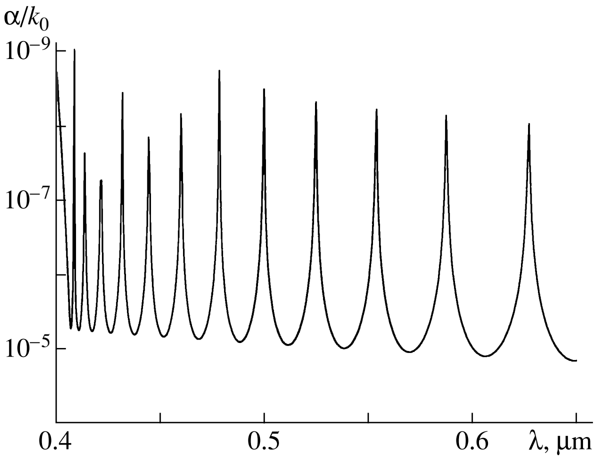

Fig. 2. Attenuation coefficient of the first leaky mode of a planar optical waveguide vs. the wavelength.

From Tables 1–5 it follows that, for the low-loss fundamental TE_1 mode, our results agree (within the accuracy of data reported in [2]) with the results obtained by the method developed in [2], including those referred to as “exact.” For the TE_2 mode, which radiates much more strongly, our results are noticeably closer to the “exact” ones.

To compare with recent more accurate data [7, 8] obtained by numerically searching for complex roots of dispersion relations (6), we analyzed the first 19 TE modes of a three-layer ARROW [7] and the first six modes of a nine-layer ARROW [7, 8]. The associated results are listed in Tables 6 and 7.

The propagation constants (effective indices) obtained with our approach are seen to completely agree (to the tenth decimal place) with the solutions to the complex dispersion equation obtained in [6, 7]. As for losses (attenuation coefficient), agreement is expectedly much worse: to the fourth–fifth decimal place for low-loss modes and to the second–third decimal place for modes with relatively high losses. However, such an accuracy in evaluating the attenuation coefficient is quite sufficient for practical purposes. The accuracy of our approach improves with decreasing attenuation because of the resonant character of wave interference in the ARROW cladding. Under reso-

Table 7

Nine-layer ARROW waveguide: $n_s = 3.5$, $n_1 = 1.46$, $n_2 = 1.50$, $n_3 = 1.46$, $n_4 = 1.50$, $n_5 = 1.46$, $n_6 = 1.50$, $n_7 = 1.46$, $n_8 = 1.50$, $n_9 = 1.46$, $n_c = 1.0$; $d_1 = 2.0$, $d_2 = 0.448$, $d_3 = 4.0$, $d_4 = 0.448$, $d_5 = 2.0$, $d_6 = 0.448$, $d_7 = 4.0$, $d_8 = 0.448$, $d_9 = 2.0$, $\lambda_0 = 0.6328$ μm

Mode	Present method		Reference [8]	
	β/k_0	α/k_0	β/k_0	α/k_0
TE_1	1.457920191	$0.0071059441 \times 10^{-4}$	1.457920191	$0.007106242 \times 10^{-4}$
TE_2	1.457791244	$0.0090529995 \times 10^{-4}$	1.457791244	$0.009053396 \times 10^{-4}$
TE_3	1.453780369	$0.1145698295 \times 10^{-4}$	1.453780369	$0.114698816 \times 10^{-4}$
TE_4	1.453045405	$0.4186909382 \times 10^{-4}$	1.453045406	$0.420121480 \times 10^{-4}$
TE_5	1.451864810	$0.6904035922 \times 10^{-4}$	1.451864807	$0.693651857 \times 10^{-4}$
TE_6	1.450269492	$0.7293545637 \times 10^{-4}$	1.450269491	$0.732515869 \times 10^{-4}$

Table 8

Four-layer Lossless waveguide: $n_s = 1.5$, $n_1 = 1.66$, $n_2 = 1.60$, $n_3 = 1.53$, $n_4 = 1.66$, $n_c = 1.0$; $d_1 = d_2 = d_3 = d_4 = 0.5$, $\lambda_0 = 0.6328$ μm

Mode	Present method		Reference [8]	
	β/k_0	α/k_0	β/k_0	α/k_0
TE_4	1.461515200	0.007027450589	1.461856641	0.007155871
TE_5	1.381626100	0.001622539422	1.382489223	0.018165877
TE_6	1.279615400	0.002901151215	1.281364436	0.035877392
TE_7	1.139829100	0.003970660680	1.142314462	0.052876075

nance, expression (14) closely approximates the power radiated by the waveguide (loss power), the sharper the resonance (in other words, the longer lived the mode considered), the closer the approximation.

In the off-resonance case, the attenuation coefficient calculated from Eqs. (11)–(13) differs significantly from that obtained with dispersion relation (9), as demonstrated by the parameters calculated for leaky modes in a four-layer planar waveguide [7] (see Table 8).

The efficiency of the method proposed is demonstrated in Fig. 2, which plots the wavelength dependence of the attenuation coefficient for the first leaky mode of a planar optical waveguide [11]. The 80-μm-thick core of this waveguide with a refractive index of 1.46 is surrounded by a cladding consisting of 52 pairs of layers 0.323 μm thick each with alternating refractive indices of 1.45 and 1.46.

CONCLUSIONS

A new approach to calculating the propagation parameters of leaky modes in planar optical waveguides and quantum semiconductor nanostructures is developed. The essence of this approach is that (i) the standard transfer matrix method is replaced by an exact finite-difference method proposed by the authors and (ii) the attenuation coefficient of leaky modes is calculated by a new approximate technique that yields values asymptotically tending to the exact ones as the attenuation decreases.

The accuracy of our method is compared with all the available data for the effective index and attenuation coefficient of ARROWs, and good agreement is observed.

Based on the new approach, a high-accuracy numerically efficient computer code is developed aimed at calculating the effective indices and attenuation coefficients of arbitrary-order leaky waves in multilayer planar optical waveguides with an arbitrary number of layers. The feasibility of accurately evaluating the performance of multilayer ARROWs is very important in view of their potential use as components of integrated optical devices.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, project nos. 04-02-16441 and 04-02-17177.

REFERENCES

1. M. A. Duguay, Y. Kokubun, T. L. Koch, and L. Pfeiffer, Appl. Phys. Lett. **49**, 13 (1986).
2. B. Liu, A. Shakouri, and J. E. Bowers, Opt. Quantum Electron. **31**, 1267 (1999).

3. W. Jiang, J. Chrostowski, and M. Fontaine, Opt. Commun. **72**, 180 (1989).
4. J. Kubica, D. Uttamchandani, and B. Culshaw, Opt. Commun. **78**, 133 (1990).
5. W. P. Huang, R. M. Shubair, A. Nathan, and Y. L. Chow, J. Lightwave Technol. **10**, 1015 (1992).
6. T. Baba and Y. Kokubun, IEEE J. Quantum Electron. **28**, 1689 (1992).
7. E. Anemogiannis, E. N. Glytsis, and T. K. Gaylord, J. Lightwave Technol. **17**, 929 (1999).
8. C. Chen, P. Berini, D. Feng, et al., Opt. Express **7**, 260 (2000).
9. M.-H. Sheng and H.-W. Chang, Appl. Opt. **44**, 751 (2005).
10. A. Mayer and J.-P. Vigneron, Phys. Rev. **59**, 4659 (1999).
11. K. M. Golant and E. I. Golant, in *Proceedings of the 10th Microoptics Conference, Jena, 2004*, on CD-ROM, paper L-63.
12. B. Kramer, Adv. Solid State Phys. **43**, 351 (2003).
13. E. I. Golant, Pis'ma Zh. Tekh. Fiz. **31** (24), 81 (2005) [Tech. Phys. Lett. **31**, 1074 (2005)].
14. M. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, 1969; Nauka, Moscow 1973).

Translated by A. Khzmalyan